Chapter 4: Downstream Entry and Forbidding Price Discrimination

Chapter 3 demonstrated that forbidding intermediate product third degree price discrimination can have harmful welfare consequences even if discrimination does not occur in the unconstrained regime: it may raise input prices throughout the downstream industry. This chapter takes the analysis one step further by demonstrating that the negative effects of forbidding price discrimination are intensified when there is free entry downstream.

4.1 N-Firm Bargaining

The Bargaining Process, Assumptions, and Notation

The single seller two buyer telegram game examined in Chapter 3 has a natural generalization to the case of N buyers. Just before time zero, before any contracts have been signed, assume that each buyer is randomly queued and labelled 1, 2,..., N according to whether it is the first, second,..., or nth buyer in line. At time zero the monopolist meets firm 1 and is randomly chosen either to propose or listen to an offer. If the offer is accepted firm 1 immediately begins producing the (downstream) monopoly output, and the monopolist decides whether to continue bargaining with firm 2 in the next period. If it continues bargaining and a second agreement is reached, firm 2 and firm 1 begin producing the 2-firm Cournot Nash equilibrium output. As the third, fourth,..., jth agreements are reached, all buyers who have signed begin producing the 3-firm, 4-firm,..., j-firm Cournot Nash equilibrium output.

If the original offer between firm 1 and the monopolist is rejected, firm 1 must go to the back of the queue, and firms are relabelled as follows: 1 becomes N, and $i \neq 1$ becomes i-1. Similarly, after j agreements have been signed the remaining firms have labels j+1, j+2,..., N. If the monopolist and the firm labelled j+1 end their meeting in disagreement, then firm j+1 becomes N, and those labelled j+k, $k \neq 1$, become j+k-1.

Just as the subgame after one buyer had signed was important in the two-firm case, so is the subgame after N-1 firms have signed important in the N-firm case. To simplify the analysis it is assumed that the monopolist makes the opening offer in this subgame, and then alternates offers with the remaining buyer until final agreement is reached. Allowing the proposer to be chosen randomly in each period in this subgame changes nothing of substance.

There are many other bargaining procedures that could be considered, and these may or may not lead to same conclusions as the one just described. It seems clear, however, that this one allows the monopolist to exercise its strongest possible credible threat: to send a firm to the back of the queue if agreement is not reached immediately. Hence, if the equilibrium of this game errs in any direction, it should err by giving the monopolist too much bargaining power.[1]

The monopolist produces a normal input, $X = \Sigma x_i$, at constant marginal and average cost c, where x_i is the amount purchased by firm i. Final product inverse demand is P(Y) where $Y = \Sigma y_i$ is total output and y_i is the output of firm i. Downstream technology, assumed freely available to all firms, is represented by the cost function

$$W(y_i, a_i, \mathbf{w}) = \begin{cases} K + V(y_i, a_i, \mathbf{w}), & y_i > 0 \\ 0, & y_i = 0 \end{cases}$$

where K is a fixed cost that is not sunk[2], V is the variable cost function, and **w** is the price vector (henceforth suppressed) of other competitively sold factors. Firms share the common discount rate $\delta = e^{-rz}$, where r is each firm's implicit rate of time preference, and z is the time between successive offers in the bargaining game. To reflect the belief that bargaining costs are small I focus on the limiting equilibrium as $z \to 0$.

Attention is restricted to downstream markets whose demand and cost conditions satisfy the following assumptions.

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[1]For example, one could allow the monopolist to choose which buyer it approaches each period. In the two buyer case this generates an additional asymmetric equilibrium in which the monopolist stays with the same buyer until that buyer signs. This equilibrium is supported by the belief held by each buyer that the monopolist will always stay until agreement is reached with that buyer. But this leads to a *worse* outcome for the monopolist since, by delaying agreement, the first buyer can effectively impose a loss on the monopolist equal to the profits it earns from both buyers.

[2]The role of sunk costs is discussed in section 4.3.

Assumption 4.1. $P(\cdot)$ is twice continuously differentiable, with $P'(Y) < 0$; there exists $\bar{Y}$ such that for all $Y \geq \bar{Y}$, $P(Y) = 0$; and $P(0) = \bar{p} < \infty$.

Assumption 4.2. $V(y,a)$ is twice continuously differentiable with with $V_y(y,a) \geq 0$, $V_{yy}(y,a) \geq 0$, $V_{ya}(y,a) > 0$, for all $y \geq 0$, $a \geq c$, $V_a(y,a) > 0$ for all $y > 0$, $a \geq c$; and $\bar{p} > V_y(0,c)$.

Assumption 4.3. For all $\mathbf{y} = (y_1\ y_2, \ldots, y_n) >> 0$, $n \geq 1$, and $a_i \geq 0$, $nP''(Y)y_i + (n + 1)P'(Y) - V_{yy}(y_i, a_i) < 0$ for all i.

Assumption 4.4. For all $\mathbf{y} >> 0$, $P'(Y) + y_i P''(Y) \leq 0$ for all i.

Assumption 4.1 says that the demand curve is downward sloping and intersects both axes. Assumption 4.2 says that marginal cost is upward sloping, x is a normal input, and monopoly production is profitable if fixed costs are small and the price of x is low. I focus on symmetric equilibrium whenever downstream firms are charged the same price for the intermediate product. Assumptions 4.1 - 4.3 guarantee that, for small enough a and K, such an equilibrium exists, is stable, and is unique among equilibria restricted to be symmetric.

Assumption 4.4 says that each firm's marginal revenue is steeper than the demand function at all output combinations. In the language of Bulow, Geanakoplos, and Klemperer (1985) this means that each firm views its output as a *strategic substitute* for that of every other firm. It implies that an increase in any one firm's output reduces the equilibrium output of each of the other firms. This assumption holds if the demand function is concave, or not too convex.

Letting $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ be the vector of prices paid for the monopolized input by the n firms in production, $y_i(\mathbf{a})$ is each firm's equilibrium output, and $x_i(\mathbf{a}) = V_a(y_i(\mathbf{a}), a_i)$ is its equilibrium input demand. The (flow) equilibrium profits of the monopolist and downstream firm i are given by

$$(4.1) \qquad U_n(\mathbf{a}) = \Sigma\ (a_i - c)\ x_i(\mathbf{a})$$

and

$$(4.2) \qquad \pi_{i,n}(\mathbf{a}) = P(Y(\mathbf{a}))\ y_i(\mathbf{a}) - W(y_i(\mathbf{a}), a_i).$$

The monopolist's incremental profit from reaching agreement with firm i when it is the nth buyer to sign is

$$(4.3) \qquad U_{i,n}(\mathbf{a}) = U_n(a_i, \mathbf{a}_{-i}) - U_{n-1}(\mathbf{a}_{-i})$$

where $\mathbf{a}_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$ and $\mathbf{a} = (a_i, \mathbf{a}_{-i})$, a notational convention followed throughout the rest of this chapter. Let

$$A_{i,n}(\mathbf{a}_{-i}) = \left\{ a_i \mid \pi_{i,n}(\mathbf{a}),\ U_{i,n}(\mathbf{a}) \geq 0;\ \frac{\partial U_{i,n}(\mathbf{a})}{\partial a_i} \geq 0 \right\},$$

the set of all input prices over which the monopolist and firm i have a conflict of interest in the subgame after firms other than i have agreed to prices $\mathbf{a}_{-i}$. This set is assumed to be convex for all i, n.

Whenever $\mathbf{a} = (a,\ldots,a)$, let $y_i(a,n) \equiv y_i(\mathbf{a})$ be the symmetric Cournot equilibrium output of each firm. This mild abuse of notational convention will be followed whenever symmetry is imposed and it causes no ambiguity.

Under Assumptions 4.1 - 4.4, the equilibrium output and profit functions satisfy several useful properties used in the derivations below. In particular, Appendix C verifies that the following properties hold in symmetric equilibrium.

<u>Property 4.1</u>. $\frac{\partial y}{\partial n} < 0, \ \frac{\partial Y}{\partial n} > 0, \ \frac{\partial x}{\partial n} < 0, \ \frac{\partial X}{\partial n} > 0.$

<u>Property 4.2</u>. $\frac{\partial \pi_{i,n}}{\partial a_i} < 0, \quad \frac{d\pi_{i,n}}{da} \to 0$ as $n \to \infty$.

<u>Property 4.3</u>. $U_{i,n} = 0$ implies that $\frac{\partial U_{i,n}}{\partial a_i} > 0$.

<u>Property 4.4</u>. $\frac{\partial \pi_{i,n}}{\partial n} < 0$; for all $a \geq 0$, for all $K > 0$ such that some N-firm equilibrium exists, there exists $N'(K,a) < \infty$ such that $\pi_{i,N'}(a) = 0$.

N-Firm Equilibrium Under Free Bargaining

The single seller two-buyer analysis of the free bargaining regime in section 3.2 generalizes in a straightforward way to the case of $N \geq$

3 downstream buyers. To see how, recall that in the two buyer case the limiting stationary equilibrium price is completely determined by what happens in the subgame after one buyer has signed a contract. That is, the monopolist (firm i) does not sign an initial contract at a price lower (higher) than it expects to receive only a short time later. Similarly, the limiting stationary equilibrium with N buyers is driven by the subgame after all but one have signed a contract. Since buyers are symmetric, the first, second,..., nth contract each converges to the same price as $z \to 0$; this price is the Rubinstein equilibrium to the subgame after N-1 buyers have signed.

Consider the subgame after all buyers except firm i have signed. If there exists a SPE to this subgame, the monopolist's offer, S, and firm i's offer, R, must satisfy

$$(4.3)\quad g(a,\mathbf{a}_{-i}) \equiv \underset{a' \in A_{i,N}(\mathbf{a}_{-i})}{\operatorname{argmax}} U_{i,N}(a',\mathbf{a}_{-i}) \text{ s.t. } \pi_{i,N}(a',\mathbf{a}_{-i}) \geq \delta\pi_{i,N}(a,\mathbf{a}_{-i}),$$

$$(4.4)\quad h(g,\mathbf{a}_{-i}) \equiv \underset{a' \in A_{i,N}(\mathbf{a}_{-i})}{\operatorname{argmax}} \pi_{i,N}(a',\mathbf{a}_{-i}) \text{ s.t. } U_{i,N}(a',\mathbf{a}_{-i}) \geq \delta U_{i,N}(g,\mathbf{a}_{-i})$$

and

$$(4.5)\quad S(\mathbf{a}_{-i}) \equiv g(h(S(\mathbf{a}_{-i}),\mathbf{a}_{-i}),\mathbf{a}_{-i}), \quad R(\mathbf{a}_{-i}) \equiv h(S(\mathbf{a}_{-i}),\mathbf{a}_{-i})$$

where $\mathbf{a}_{-i}$ is the vector of prices agreed to by all firms other than i. Focusing on situations where bargaining costs are small, bargaining equilibrium is defined as follows.

Definition 4.1. Let $S(\mathbf{a}_{-i},z)$ be the monopolist's equilibrium offer in the subgame after all but firm i have signed, explicitly accounting for its dependence on z. An *N-firm equilibrium under free bargaining* is a price a^0 (which depends on N) such that $a^0 = \lim_{z \to 0} S(a^0,\ldots,a^0,z)$.

Binmore (1986a) demonstrated that as $z \to 0$, equilibrium to the subgame (as defined by equations (4.3) - (4.5)) approaches the Nash bargaining solution (with threat points set equal to zero) if the frontier of the set

$$\{ (U_{i,N},\pi_{i,N}) \mid a_i \in A_{i,N}(\mathbf{a}_{-i}) \}$$

is concave. However, since the curvature of the frontier generally depends on the third derivative of the demand function, concavity is not guaranteed by Assumptions 4.1 - 4.4. If it is not concave, the Nash bargaining solution is not even defined.[3] The following proposition establishes a convenient characterization of the limiting equilibrium that does not require the frontier to be concave.

Proposition 4.1. Let a^0 be an N-firm equilibrium under free bargaining, and let $\mathbf{a}^0 = (a^0,\ldots,a^0)$. Then, for all $i \in \{1,\ldots,N\}$,

[3]The usual way to get around this problem is to allow agents to randomize over the set of possible outcomes. It is difficult, however, to interpret this kind of randomization in a game where agents explicitly exchange offers.

$$(4.6)\qquad \frac{\partial U_{i,N}(a^0)}{\partial a_i}\pi_{i,N}(a^0) + \frac{\partial \pi_{i,N}(a^0)}{\partial a_i}U_{i,N}(a^0) = 0.$$

<u>Proof</u>: First I argue that the constraints in problems (4.3) and (4.4) bind in equilibrium for z close to zero. Suppose the constraint in problem (4.4) does not bind. Then for all $z > 0$, firm i can reduce its offer below R by a small amount while still satisfying the constraint. But by Property 4.2, this increases firm i's profits, contradicting the definition of R.

Next, suppose that for all $\omega > 0$, there exists some $z' \in (0,\omega]$ such that the constraint in problem (4.3) does not bind. This implies that there is a sequence $\{z_t\} \to 0$ such that $\partial U_{i,N}(S(a_{-i},z_t),a_{-i})/\partial a_i = 0$ for all $z_t < \omega$, and therefore that $\partial U_{i,N}(S(a_{-i},z_t),a_{-i})/\partial a_i \to 0$. I will show that this yields a contradiction.

In equilibrium, $U_{i,N}$ is non-decreasing in a neighborhood below both S and R; otherwise both the monopolist and firm i would want to reduce price. Since the constraint in problem (4.4) binds, this implies that $S > R$ for z close to 0. Furthermore, by the continuity of $U_{i,N}$ and the fact that the constraint in problem (4.4) binds, either $U_{i,N} \to 0$, or $S \to R$ as $z \to 0$. But $U_{i,N} \to 0$ and Property 4.3 together imply that $\partial U_{i,N}/\partial a_i \not\to 0$, yielding a contradiction.

Suppose, then, that $S \to R$. Expanding the right hand side (RHS) of each constraint about the price on the LHS of that constraint and rearranging the resulting expressions yields

$$(4.7) \qquad \frac{-\pi_{i\ N}(S,a_{-i})}{\partial\pi_{i\ N}(v,a_{-i})/\partial a_i} > \frac{\delta}{(1-\delta)}(S-R)$$

and

$$(4.8) \qquad \frac{\partial U_{i,N}(t,a_{-i})/\partial a_i}{U_{i,N}(R,a_{-i})} = \frac{(1-\delta)}{\delta}\frac{1}{(S-R)}$$

where t, v ϵ [R,S]. Since $S \rightarrow R$, t and v also converge to the same price, call it $\bar{a}$. Multiplying equations (4.7) by (4.8) and taking the limit as $z \rightarrow 0$ implies $0 > 1$, which is a contradiction. Therefore, there exists ω such that the constraint binds for all z ϵ $(0,\omega]$.

It remains to show that equation (4.6) is satisfied for all such z. Change the inequality in equation (4.7) to an equality. If both $U_{i,N}(\bar{a},a_{-i})$ and $\pi_{i,N}(\bar{a},a_{-i})$ equal zero, then equation (4.6) is satisfied trivially. If neither, or one of them equals zero, then (4.7) and (4.8) can be rearranged and divided (without dividing by zero) to yield equation (4.6). Q.E.D.

From equation (4.6) it is not immediately obvious how the equilibrium price varies with N. However, there is an immediate corollary describing equilibrium for any given number of buyers.

<u>Corollary 4.1</u>. For all $N < \infty$ such that an N-firm equilibrium exists, the monopolist does not exercise price leadership in an N-firm equilibrium under free bargaining.

Proof. This follows from the demonstration in the proof of Proposition 4.1 that $\partial U_{i,N}/\partial a_i \not\to 0$.

Example: Bargaining Power and the Number of Downstream Firms

One might expect, quite naturally, that most plausible measures of the monopolist's bargaining power would rise with the number of downstream firms. However, a simple example shows that this intuition is wrong. Suppose $P(Y) = \alpha - Y$, $W(y,a) = ay$ ($K = 0$, $x = y$), and $c = 0$. Then, the equilibrium output of each downstream firm is $y = (\alpha - a)[1/(N+1)]$, and straightforward calculations yield

$$(4.9) \qquad U_i(a,\ldots,a,N) = \frac{(\alpha - a)a}{N(N+1)},$$

$$(4.10) \qquad \frac{\partial U_i(a,\ldots,a,N)}{\partial a_i} = \frac{\alpha - 2a}{N+1},$$

$$(4.11) \qquad \pi_i(a,\ldots,a,N) = \frac{(\alpha - a)^2}{(N+1)^2},$$

and

$$(4.12) \qquad \frac{\partial \pi_i(a,\ldots,a,N)}{\partial a_i} = \frac{-2N(\alpha - a)}{(N+1)^2}.$$

Substituting these expressions into equation (4.6) and solving for equilibrium price yields $a^0 = \alpha/4$, independently of N. Hence, entry has no effect on the equilibrium input price when final product demand is linear and firms have constant marginal and average costs.

Although it may have been unexpected, this result is really quite intuitive. When bargaining costs are small, each firm's bargaining power is proportional to the size of the incremental loss it can impose (by delaying service) on each firm with which it negotiates. As the number of buyers increases, the loss each buyer can impose on the monopolist, U_i, falls, but so does the loss that the monopolist can impose on each buyer. Entry transfers bargaining power to the monopolist only if the profit earned by each buyer weighted by the slope of the monopolist's profit function grows relative to seller's (symmetrically weighted) incremental profit from selling to that buyer. It turns out, in the linear case, that entry changes these functions such that equilibrium input price is held constant.

It is clear why the apparent advantage the monopolist holds early in negotiations is really no advantage at all: each buyer can credibly threaten to reject high prices in favor of waiting for a more symmetric bargaining position after a very short delay. There are two ways the monopolist could improve its plight. One way would be to take some action credibly committing itself not to bargain so symmetrically with buyers signed in later periods. Alternatively, it could try to change the rules of the game to eliminate buyers' credible threats. The next section examines how a rule forbidding price discrimination does precisely this.

Forbidding Price Discrimination

Generalizing from the single seller two buyer case when price discrimination is forbidden proceeds along the lines of the derivation in section 3.3. Let a^* and $\hat{a}$ be the SSPE offers of the monopolist and each buyer in every period before any contracts have been signed. In equilibrium, firm 1 (resp. the monopolist) must be indifferent between accepting and rejecting a^* (resp. $\hat{a}$) when the time between offers is sufficiently close to zero.[4] The equations reflecting these indifference constraints when there are N buyers are

$$(4.13) \quad (1 - \delta)\, \pi_{1,1}(a^*) + (\delta - \delta^2)\, \pi_{1,2}(a^*) + \ldots + (\delta^{N-2} - \delta^{N-1})\, \pi_{1,N-1}(a^*) + \delta^{N-1}\, \pi_{1,N}(a^*) = \frac{\delta^N}{2} \left[\pi_{1,N}(a^*) + \pi_{1,N}(\hat{a}) \right],$$

and

$$(4.14) \quad (1 - \delta)\, U_1(\hat{a}) + (\delta - \delta^2)\, U_2(\hat{a}) + \ldots + (\delta^{N-2} - \delta^{N-1})\, U_{N-1}(\hat{a}) + \delta^{N-1}\, U_N(\hat{a}) = \frac{\delta}{2} \left[(1 - \delta)\, U_1(\hat{a}) + \ldots + \delta^{N-1}\, U_N(\hat{a}) + (1 - \delta)\, U_1(a^*) + \ldots + \delta^{N-1}\, U_N(a^*) \right],$$

[4] The demonstration that these indifference constraints bind for z close to zero follows a line of argument similar to that in Proposition 4.1, and hence, is omitted.

where $\mathbf{a}^* = (a^*,\ldots,a^*)$ and $\hat{\mathbf{a}} = (\hat{a},\ldots,\hat{a})$.

Expanding the RHS of each equation about the price on the LHS of that equation yields

$$(4.15)\qquad (1-\delta)\,\pi_{1,1}(\mathbf{a}^*) + (\delta-\delta^2)\,\pi_{1,2}(\mathbf{a}^*) + \ldots + (\delta^{N-1}-\delta^N)\,\pi_{1,N}(\mathbf{a}^*) = \frac{\delta^N}{2}\,\frac{d\pi_{1,N}(\mathbf{t})}{da}\,(a^*-\hat{a}),$$

and

$$(4.16)\qquad (1-\delta)\left[\,(1-\delta)\,U_1(\hat{\mathbf{a}}) + (\delta-\delta^2)\,U_2(\hat{\mathbf{a}}) + \ldots + (\delta^{N-2}-\delta^{N-1})\,U_{N-1}(\hat{\mathbf{a}}) + \delta^{N-1}\,U_N(\hat{\mathbf{a}})\,\right]$$
$$= \frac{\delta}{2}\left[(1-\delta)\,\frac{dU_1(\mathbf{v})}{da} + (\delta-\delta^2)\,\frac{dU_2(\mathbf{v})}{da} + \ldots + (\delta^{N-2}-\delta^{N-1})\,\frac{dU_{N-1}(\mathbf{v})}{da} + \delta^{N-1}\,\frac{dU_N(\mathbf{v})}{da}\right](a^*-\hat{a})$$

for some t, $v \in [\hat{a},a^*]$, where $\mathbf{t} = (t,\ldots,t)$, and $\mathbf{v} = (v,\ldots,v)$. As $z \to 0$, the LHS of (4.15) goes to zero, implying that $\hat{a} \to a^*$. Hence, as $z \to 0$, $\hat{a}$, a^*, t, and v all converge to the same price, say a^F. Dividing (4.15) by (4.16) and taking limits yields[5]

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[5]If dU_N/da or U_N goes to zero, the equations are divided in the way that avoids dividing by zero.

$$\text{(4.17)} \qquad \frac{dU_N}{da}(\pi_{1,1} + \pi_{1,2} + \ldots + \pi_{1,N}) + \frac{d\pi_{1,N}}{da} U_N = 0.$$

Definition 4.2. An *N-firm equilibrium when price discrimination is forbidden* is a price, a^F, satisfying equation (4.17).

An interesting connection between the asymmetric Nash bargaining solution and this equilibrium facilitates its interpretation. Assuming that the frontier of the set

$$\{ (\pi_{1,N}, U_N) \mid a_i \in A_{i,N}(a_{-i}),\ a_i = a_j \ \forall\ i, j \}$$

is concave, an asymmetric Nash bargaining solution with threat points set equal to zero can be defined as a price solving

$$\text{(4.18)} \qquad \max \left\{ \pi_{1,N}(a)^{\lambda}\, U_N(a)^{1-\lambda} \mid a_i \in A_{i,N}(a_{-i}),\ a_i = a_j \ \forall\ i, j \right\}$$

where $\lambda / (1 - \lambda)$ measures the bargaining power of firm 1 relative to that of the monopolist. The first order necessary (and sufficient) condition for an interior solution is

$$\text{(4.19)} \qquad \frac{dU_N}{da} \pi_{1,N} + \frac{\lambda}{1 - \lambda} \frac{d\pi_{1,N}}{da} U_N = 0.$$

Now, let $\lambda = \pi_{1,N} / (\pi_{1,1} + \pi_{1,2} + \ldots + \pi_{1,N-1} + 2\pi_{1,N})$, which implies that $\lambda/(1 - \lambda) = \pi_{1,N} / (\pi_{1,1} + \pi_{1,2} + \ldots + \pi_{1,N})$ is firm 1's relative bargaining power. Then equation (4.19) reduces to equation (4.17).

Hence, N-firm equilibrium when price discrimination is forbidden can be interpreted as an asymmetric Nash bargaining solution where firm 1's equilibrium relative bargaining power is determined simultaneously with the input price.

This also leads to the following intuitive comparative statics result. Holding the input price constant, an increase in N decreases $\pi_{1,N}$ and increases $\pi_{1,1} + \cdots + \pi_{1,N}$, leading to a decrease in firm 1's relative bargaining power. This reduces the first order condition (4.19), and therefore raises the equilibrium price. Hence, an increase in the number of downstream firms reduces firm 1's relative bargaining power leading to an increase in the equilibrium price.

In the free bargaining example of the last section the monopolist's bargaining power did not rise with the number of downstream firms. Why are these results different? The reason is that when price discrimination is forbidden, buyer bargaining power no longer derives from the ability to impose incremental losses on the monopolist; rather, it derives from the monopolist's ability to play buyers against each other to determine *the* initial price. The easiest way to see this is in terms of the two phases of the bargaining game in which forbidding price discrimination affects negotiations. Phase one is the bargaining that occurs before any initial agreements have been signed; phase two is the bargaining that occurs after the first agreement has been signed. In the unconstrained regime, phase two bargaining is not constrained. Buyers can therefore wait to be the last to sign a contract, at which point they have about the same bargaining power as the monopolist. In contrast, forbidding price discrimination effectively disallows phase two bargaining. This means that all bargaining occurs in phase one. But the ability of the monopolist to play buyers against each other in phase one increases with the number of buyers as the threat to send disagreeable buyers to the end of the queue carries more force. This allows the monopolist to demand a higher price in phase one.

4.2. Free Entry

Assume there are a large number of firms contemplating entry in the downstream industry. The entry process is modelled by breaking the game into two stages. In the first stage N firms enter, in the second stage N-firm equilibrium is determined. Ignoring the integer constraint, firms are required to earn zero profits in free entry equilibrium.

Definition 4.3. An *equilibrium with free entry under free bargaining* is a price $a^0(K)$ and a number $N^0(K)$ such that $\pi_{i,N^0}(a^0,...,a^0) = 0$, and a^0 is an N^0-firm equilibrium under free bargaining.

Definition 4.4. An *equilibrium with free entry when price discrimination is forbidden* is a price $a^F(K)$ and a number $N^F(K)$ such that $\pi_{1,N^F}(a^F,...,a^F) = 0$, and a^F is an N^F-firm equilibrium when discrimination is forbidden.