

# Tying, Bundling, and Double Marginalization

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## Abstract

We provide a general definition of bundling that encompasses the bundling of two or more objects over sets of three or more objects. Bundled objects may be units of the same product, different products, or both. Such bundling encompasses a range of controversial pricing practices that have drawn antitrust scrutiny in recent years. By nesting these practices in a common framework we are able to analyze their microfoundations. We find that the pricing inefficiency generally known as “double marginalization” arises not from the absence of inframarginal transfers, but from the inability to bundle objects over which the buyer experiences declining incremental benefits. Whether the objects are units of the same product or different products makes no difference. Thus, we identify an efficiency benefit of tying and bundling that has been missed in the literature. We discuss the implications of our findings for public policy toward these practices.

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Pricing terms between adjacent firms in supply chains often involve a complex form of bundling. Volume commitments and discounts, which are common, effectively bundle the units of specific products. Package deals of various types, which are also common, effectively bundle the units of different products. In multi-product settings, both forms of bundling can occur simultaneously.

In this paper, we nest these practices in a common framework and analyze their microfoundations. Our analysis is motivated by two observations. First, the literature on bundling has focused largely on the practice of selling bundles of products in fixed proportions at a common price to multiple buyers. However, this is not the form bundling takes in many intermediate good markets, where terms are often buyer-specific and involve more complex bundling. Second, antitrust concerns related to tying and bundling arise most often in intermediate good markets. The classic bundling literature is inadequate for addressing these concerns because it assumes fixed proportions bundling and linear pricing for products that are not part of the bundle. Although some of the tying literature addresses the variable proportions case, that literature also tends to assume linear pricing for the individual products that are tied. Neither literature captures the range of generalized bundling strategies that appear in contracts between adjacent firms in supply chains.

From a policy perspective, the antitrust treatment of tying and bundling in intermediate-goods markets is highly unsettled, and the lack of consistency in treatment across practices that appear at their core to involve different forms of bundling is striking. For example, tying is illegal in the U.S. when it involves two or more products, the seller has sufficient market power over the tying product, and the tie-in affects a not insubstantial amount of commerce in the tied product market.<sup>1</sup> Bundled discounts, by contrast, are usually lawful unless the discounts involve below cost pricing.<sup>2</sup> Quantity discounts, a variant of bundled discounts that conditions the price of each unit of a product on whether other units are purchased, are rarely condemned by antitrust authorities.<sup>3</sup>

The disparity in the legal treatments raises obvious questions. All three strategies – tying, bundled discounts, and quantity discounts – appear to have elements of bundling, suggesting at an intuitive level that they likely have at least some common effects. The primary anticompetitive

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<sup>1</sup>See American Bar Association (2007). Although tying enforcement has become more lenient over the years and the U.S. Department of Justice has argued that tying should not be per se illegal (U.S. Department of Justice, 2008, p. 87), the Supreme Court has not overturned the structured per se standard set out in this three step test. In its most recent statement on the matter, the Supreme Court stated “[W]e have condemned tying arrangements when the seller has some special ability—usually called ‘market power’—to force a purchaser to do something that he would not do in a competitive market. . . .” *Illinois Tool Works Inc. v. Independent Ink, Inc.*, 547 U.S. 28 (2006), p. 7, reiterating language from *Jefferson Parish Hospital Dist. No. 2 v. Hyde*, 466 U. S. 2.”

<sup>2</sup>See generally the discussion in U.S. Department of Justice (2008), pp. 91-95.

<sup>3</sup>The main exception is *all-units* quantity discounts, which have been condemned both in the U.S. and Europe as exclusionary in a handful of cases. Examples include the FTC’s case against Intel (Complaint, *In re Intel Corp.* (FTC Dec. 16, 2009) (No. 9341)) and the European case against Tomra (case COMP/E-1/38.113 - Prokent-Tomra).

concern for tying is that it may discourage entry or investment by competitors in either the tying or the tied product markets (Whinston, 1990; Carlton and Waldman, 2002). It is not clear, however, why bundled discounts would not raise the same concern, or why quantity discounts would not raise an analogous concern — discouraging entry or investment by rival suppliers of different units of the same product. Why, then, are the different practices treated so differently by the law?

One distinction that is sometimes made to justify the pro-competitive treatment of quantity discounts is that they can be used to mitigate double marginalization (Spengler, 1950). In contrast, the welfare implications of tying and bundling when they are not used for foreclosure purposes are thought to be ambiguous.<sup>4</sup> But again, this leaves an important question unanswered: why does the “tying” of different units of a single product via quantity discounts have benefits that do not arise from the tying of different products? The only analytical difference between these cases would seem to be the relationship between the objects that are tied (units of the same product versus units of different products), and it is not obvious why this difference would justify disparate treatment.

Surprisingly, the distinction between bundling units and bundling products not been systematically addressed in the economic literatures on either tying or bundling. This gap likely accounts for some of the ongoing controversy over whether bundled discounts should be treated relatively aggressively like tying or more passively like quantity discounts. And it may help to explain the colorful language that some have used to describe the current state of the law on tying and bundling.<sup>5</sup>

The bundling framework we develop offers a unified treatment for evaluating these issues.<sup>6</sup> Specifically, we envision a supplier selling one or more products to a downstream buyer who uses the products to produce one or more final products. The supplier’s pricing problem is to set prices for every unit it offers for sale and every possible bundle of units it offers for sale to maximize its profit given the buyer’s optimal purchase decision and any limitations on the supplier’s ability to bundle. We show that different bundling practices emerge as equilibrium strategies under different limitations on bundling, limitations may arise from legal constraints, transactions costs, or both.

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<sup>4</sup>The price discrimination effects of bundling and tying are thought to include metering (Bowman, 1957), and reducing the heterogeneity of consumer valuations through bundling (Adams & Yellen, 1976). It is well-known that both strategies can increase or decrease welfare, depending on demand and cost conditions.

<sup>5</sup>See, for example, Jaeckel (2010), “LePage’s, Cascade Health Solutions, and a Bundle of Confusion;” and Evans (2006), “Tying: The Poster Child for Antitrust Modernization.”

<sup>6</sup>We study practices in which a seller’s prices depend on its customers’ purchases of its own products but not their purchases of rivals’ products. This class of practices includes non-requirements tying, bundled discounts, quantity discounts, and variations on these strategies (e.g., full-line forcing). This class does not include requirements tying, which imposes the condition that the buyer take all of its needs for the tied product from the tying product seller. Requirements tying is more akin to exclusive dealing, which requires the buyer to take all of its needs for a specific product from the seller as a condition for purchasing any amount of that product from the same seller. Requirements tying is thus similar to exclusive dealing, but it conditions the exclusivity for the tied product on the purchase of any amount of the tying product from the same seller.

Our analysis exposes a fundamental efficiency property of non-requirements tying and bundled discounts that has been missed in literature. Specifically, we show that when tying or bundled discounts are profitable strategies in bilateral exchange, the efficiency benefits that are generated are isomorphic to the benefits of using nonlinear pricing to eliminate double marginalization. This finding has far-reaching implications. It suggests that when contracts are buyer-specific, non-requirements tying, bundled discounts, and nonlinear pricing should receive similar legal treatment.

The benefits of bundling arise in two forms: an increase in the quantities of the products that are sold, and an increase in product variety. Just as the ability to bundle units of a particular product causes an increase in the quantity sold of that product (by eliminating double marginalization), the ability to bundle across units of different products leads to an increase in the quantities of the bundled products that are sold. It may also lead to an increase in the number of products sold.

Understanding the link between these effects and double marginalization requires tracing the source of the pricing inefficiency that arises in distribution channels when buyers and sellers both have market power. In doing so, we identify the core intuition behind the double marginalization that arises in the single-product case as well as in our multi-product extension. The canonical intuition for double marginalization in the single-product case is that in the absence of fixed fees (or other inframarginal payments) to transfer surplus, an upstream seller will distort its marginal transfer prices in an effort to capture surplus. This intuition has its place, but it misses the key issue. For example, the ability to bundle units through the nonlinear pricing of specific products gives the supplier access to inframarginal transfers, but our results demonstrate that this is not sufficient to support the efficient outcome. Clearly, the standard intuition is missing a key element.

To understand the core intuition, suppose the supplier cannot bundle at all. Then, the maximum price the supplier can charge for a particular unit is constrained by the buyer's incremental profit from purchasing that unit, as the buyer can always threaten to drop it. The buyer's incremental profit from a particular unit, say A, generally depends on the prices of other units, as these prices affect the buyer's profit from dropping unit A. The key factor behind inefficient pricing when bundling is infeasible is this: by raising the prices of units *other than* unit A, the supplier relaxes constraints that prevent it from raising the price of unit A. This is true for the pricing of each and every unit, whether the units are homogeneous or differentiated. Our results imply that this key factor is present in a bilateral relationship whenever bundling is profitable, which is true whenever the buyer has declining incremental profit over any set of units. In the single-product case with continuous quantities, declining incremental profit is simply concavity of the buyer's profit, and

the inefficiency that arises is known as double marginalization. The inefficiency occurs because for each unit A, raising the prices of all units other than unit A relaxes the constraints that prevent the supplier from raising the price of unit A. The same argument explains why constraints on bundling in our setting yield analogous inefficiencies, which take the form of lower variety and/or quantity.

An interesting corollary emerges for the single-product case that sheds additional light on the role of bundling in optimal pricing. Under declining incremental profit, when bundling would be profitable if allowed, we show that the price of each unit sold in the absence of bundling must equal the buyer's gross incremental profit (excluding prices) from the unit. Because the gross incremental profit is the same for each unit of a particular product, every unit sold of a given product must therefore have the same price, i.e., the seller must offer a *linear* price for each product sold. This demonstrates that linear pricing is a necessary consequence of a supplier's inability to bundle.

Linear pricing is loosely known as the source of double marginalization in the economic folklore. Our analysis, however, traces the source of double marginalization to the infeasibility of bundling and shows that linear pricing is a consequence of the same underlying factor. Another substantive difference from the folklore is that the absence of bundling does not imply the absence of nonlinear pricing. Quantity premia, for example, are a form of nonlinear pricing that need not involve bundling. Quantity discounts do involve bundling, however, and because the type of nonlinear pricing that is needed to mitigate double marginalization entails quantity discounts, banning bundling leads to double marginalization. That the key issue is the feasibility of bundling rather than the feasibility of nonlinear pricing is also apparent from our findings that the inefficiency from the infeasibility of bundling units in the single product case extends to the inability to bundle different products in the multiproduct case. A form of double marginalization arises in both cases.

The formal economic literature on tying and bundling has two main themes: the exclusion of competitors (Whinston, 1990; Carlton and Waldman, 2002) and the extraction of surplus through price discrimination.<sup>7</sup> To this we add a third motive, mitigating double marginalization. The rationale for this is that tying and bundling can arise from the same economic forces that lead to double marginalization in the single product case. This explanation is unrelated to the other price-discrimination motives for bundling in the literature, which consider environments with heterogeneous buyers that the supplier cannot distinguish. Metering refers to variable-proportions ties in which the supplier collects more from more intense users (Bowman, 1957). Metering has no role

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<sup>7</sup>Tying has many other explanations. A useful survey is Tirole (2005), which lists distribution cost savings, compatibility cost savings, information and liability considerations, protection of intellectual property, and legitimate price response, as possible additional motivations for tying. See also Evans and Salinger (2005).

in our analysis because tariffs in our model are buyer-specific. For the same reason, our explanation also differs from the classical notion of bundling that reduces the heterogeneity of buyer valuations (e.g., Adams and Yellen, 1976; McAfee et al., 1989; Fang and Norman, 2006; Chu et al., 2011).

The bundling problem we examine is formally a multi-product mechanism design problem. Several authors have examined variants of this problem under the standard assumption in the mechanism design literature that the seller does not know the buyer’s valuation.<sup>8</sup> Manelli and Vincent (2006) explore optimal mechanisms for a firm that has  $n$  objects to sell to a single buyer. They find conditions under which bundling is optimal. Our paper differs from theirs and the rest of the multi-product mechanism design literature in two important ways. First, we assume complete information, which simplifies the problem and leads to much stronger results. Second, we explore the implications of exogenous constraints on the ability of the seller to bundle units of the same product and across units of different products, as these constraints are relevant for policy analysis.

The remainder of this paper is organized as follows. Section I presents the formal economic framework. Section II characterizes equilibrium when bundling is infeasible. Section III considers the important case where bundling is feasible over some units but not others. Section IV considers the case of increasing incremental profits (e.g., complements). Section V concludes the paper. The Appendix contains all formal proofs not presented in the main body of the paper.

## I. Economic Framework

### A. Production, Pricing, and Profits

An upstream firm (the “supplier”) sells its output to a downstream firm (the “buyer”) who resells the output to final consumers.<sup>9</sup> The supplier can produce a finite set of objects  $X = \{x_1, x_2, \dots, x_U\}$ . The objects in  $X$  represent the units of one or more distinct products. For example, objects  $x_1, \dots, x_U$  might represent different units of a single product. Alternatively, the objects might represent single units of different products. Or, objects  $x_1, \dots, x_k$  might represent units of one product, and objects  $x_{k+1}, \dots, x_U$  might represent units of one or more other products, etc. We place no restrictions on  $X$  beyond the assumption that it is finite.

The supplier’s price schedule assigns prices to all possible combinations of objects in  $X$ . This schedule is given by  $p: 2^X \rightarrow R_+$ , where  $2^X$  is the set of all subsets of  $X$ . We assume that the price

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<sup>8</sup>Examples include McAfee and McMillan (1988), Wilson (1993), Armstrong (1996, 1999), Rochet and Chone (1998), Wilson (1993), Manelli and Vincent (2006, 2007). Roche and Stole (2003) provide a survey.

<sup>9</sup>The assumption that output is resold by the buyer rather than used as an input into further production is an unimportant simplification made purely for convenience.

of purchasing nothing is zero,  $p(\emptyset) = 0$ . Given the supplier's price schedule  $p(\cdot)$ , the buyer decides which objects to purchase, the supplier fills the buyer's order, and all payoffs are realized.

Let  $S \subseteq X$  be the set of objects purchased. Then, the supplier's profit is  $p(S) - C(S)$ , where  $C(S)$  is the supplier's cost of producing  $S$ , and the buyer's profit is  $\Pi_B(S) - p(S)$ , where  $\Pi_B(S)$  is the buyer's net profit in the downstream market. We allow for the possibility that the buyer may not want to resell all the objects in  $S$ , and we assume that profit functions are common knowledge.<sup>10</sup>

The following definition provides the key conditions on the buyer's profit that drive our results:

**Definition 1** *Declining, Increasing, and Constant Incremental Profit*

(a) *The buyer has "declining incremental profit" over the set of objects  $Z \subseteq X$  if and only if*

$$\Pi_B(S' + S'' + \Delta S) - \Pi_B(S' + S'') \leq \Pi_B(S' + \Delta S) - \Pi_B(S')$$

*for all non-overlapping sets  $S'$ ,  $S''$ ,  $\Delta S$ , that are subsets of  $Z$ .*

(b) *The buyer has "incremental profit" if the inequality in (a) is reversed.*

(c) *The buyer has "constant incremental profit" if the condition in (a) holds with equality.*

This says that when the buyer's incremental profit is declining, the change in the buyer's net profit when it buys additional objects  $\Delta S$  is weakly smaller the more objects the buyer is already purchasing. The opposite is true when the buyer's incremental profit is increasing.

An economic interpretation of declining incremental profit is that the objects in  $Z$  are substitutes from the buyer's perspective in the sense that the buyer's benefit from purchasing additional objects from the seller declines with the total number of objects purchased. In mathematical terms, this means that  $\Pi_B(S)$  is a submodular set-valued function. Similarly, an economic interpretation of increasing incremental profit is that the objects  $Z$  are complements from the buyer's perspective in the sense that the buyer's benefit from purchasing additional objects from the seller increases with the total number of objects purchased. In this case,  $\Pi_B(S)$  is supermodular. The phrase "from the buyer's perspective" is needed because some or all of the objects in  $S$  may, for example, be independent in the eyes of consumers but nevertheless still be complements or substitutes from the buyer's perspective if there are sufficient economies or diseconomies of scale and scope in resale.

The primitives that underlie whether a buyer's incremental profit is declining or increasing thus depend not only on the nature of final demands and the buyer's downstream pricing behavior, but

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<sup>10</sup>Whether the buyer purchases more than it intends to resell depends, of course, on the specification of  $p(\cdot)$ .

also on the buyer’s distribution costs. For example, in the case of a single product, when distribution costs are linear and the price to consumers is the same for each unit sold, the buyer’s incremental profit is declining if and only if final demand is not too convex. In the case of multiple products, declining incremental profit (submodularity) holds if the products are weak gross substitutes in derived demand. Weak gross substitutes holds under constant marginal costs of distribution if the products are substitutes or independent, and it may hold if the products are complements but incremental costs are increasing. Declining incremental profit does not, however, imply that the derived demands are weak gross substitutes.

## B. Definition of Bundling

The classical bundling literature (Stigler, 1968; Adams and Yellen, 1976; McAfee et al., 1988) focuses on the effects of combining two objects (e.g., two single-unit products), and the objects are said to be bundled if the price the buyer pays for the objects differs from the sum of the individual prices. This definition is inadequate when there are more than two objects because it does not distinguish between cases where the supplier bundles some objects but not others from a case where the supplier bundles all of the objects. For example, suppose there are three objects, A, B, and C. Then, merely knowing that the combined price of A, B, and C differs from the sum of the individual prices does not tell us anything about whether all three objects are bundled, or only two of the objects are bundled. Nor does focusing exclusively on pairwise comparisons of objects get us very far because whether one concludes that A and B, for example, are bundled may well depend on whether the buyer also purchases C.<sup>11</sup> That is, given  $p(\cdot)$ , the combined price of A and B may equal the sum of the individual prices of A and B when C is not purchased, but may not equal the sum of the individual prices when C is purchased, and vice versa. In addition, the classical literature invariably assumes that the bundled objects are units of different products and does not allow for the possibility that the bundled objects might be different units of the same product. This is also inadequate because suppliers often bundle the units of particular products (e.g., with nonlinear pricing), sometimes at the same time that they bundle the units of different products.

With these observations in mind, we define bundling as follows:

**Definition 2** *Bundling*

*Let  $S$ ,  $S'$ , and  $S''$  be non-overlapping subsets of  $Z \subseteq X$ .*

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<sup>11</sup>We thank Glen Weyl for pointing this out.

(a) The set of objects  $S$  is not bundled with the set of objects  $S'$  conditional on  $S''$  if and only if

$$p(S + S' + S'') - p(S'') = [p(S + S'') - p(S'')] + [p(S' + S'') - p(S'')]. \quad (1)$$

(b) The set of objects  $S$  is not bundled with the set of objects  $S'$  over  $Z$  if and only if  $S$  is not bundled with  $S'$  conditional on  $S''$  for all  $S''$  in  $Z$  (where  $S''$  may be the empty set).

(c) The set of objects  $S$  is bundled with the set of objects  $S'$  conditional on  $S''$  if condition (a) does not hold; and similarly  $S$  is bundled with  $S'$  over  $Z$  if condition (b) does not hold.

(d) The objects in  $S$  are unbundled over  $Z$  if no subset of  $S$  is bundled over  $Z$  with any non-overlapping subset of  $Z$ .

This definition is flexible enough to allow for the possibility that whether  $S$  is bundled with  $S'$  depends on what else the buyer may be purchasing. We will give examples later to show why this distinction is important to make, but for now we simply note in words that our definition implies that  $S$  and  $S'$  are unbundled over  $Z$  conditional on  $S''$  if and only if the following is true: conditional on purchasing  $S''$ , the additional payment from purchasing  $S$  and  $S'$  together with  $S''$  equals the additional payment from purchasing each of  $S$  and  $S'$  in separate transactions with  $S''$ . Further, it implies that  $S$  and  $S'$  are unbundled over  $Z$  if and only if they are unbundled conditional on all possible sets of other objects  $S''$  in  $Z$ . Observe that if the buyer purchases only  $S$  and  $S'$  (i.e., if  $S'' = \emptyset$ ), then condition (1) becomes  $p(S + S') = p(S) + p(S')$ . When  $S$  is interpreted as a single unit of one product and  $S'$  as a single unit of another, this is the classical definition of no bundling. Lastly, note that the objects in  $S$  are unbundled with each other and more generally over  $Z$  if no subset of objects in  $S$  is bundled with any non-overlapping subset of objects in  $Z$ .

Our definition generalizes the classical definition to cover cases in which the seller can condition prices on all possible sets of objects. It covers bundling units of the same product and of different products and allows these practices to occur in any combination.

To illustrate, suppose once again that the set  $X$  consists of three objects, A, B, and C. Let  $p_i = p(\{i\})$  be the price if the buyer purchases only object  $i$ ,  $p_{ij} = p(\{i, j\})$  be the price if the buyer purchases objects  $i$  and  $j$ , and  $p_{ijk} = p(\{i, j, k\})$  be the price if the buyer purchases all three. Consider the following three tariffs, which illustrate the relationship between our definition and certain tariffs observed in practice.

*Tariff 1:* Objects A, B, and C are each bundled with the others.

$$p_A = \$11, p_B = \$11, p_C = \$11; \quad p_{AB} = \$12, p_{AC} = \$12, p_{BC} = \$12; \quad p_{ABC} = \$15.$$

*Tariff 2:* Objects A and B are bundled, but object C is not bundled with A, B, or  $A \cup B$ .

$$p_A = \$10, p_B = \$10, p_C = \$5; \quad p_{AB} = \$15, p_{AC} = \$15, p_{BC} = \$15; \quad p_{ABC} = \$20.$$

*Tariff 3:* No object is bundled with any other object.

$$p_A = \$10, p_B = \$11, p_C = \$12; \quad p_{AB} = \$21, p_{AC} = \$22, p_{BC} = \$23; \quad p_{ABC} = \$33.$$

Under Tariff 1, each object is bundled with each other object ( $p_{ij} < p_i + p_j$  for all  $i \neq j$ ), and each object is bundled with the set of all other objects ( $p_{ijk} < p_{ij} + p_k$  for all  $i \neq j \neq k$ ). This type of bundling is common in practice. If A, B, and C are (single-unit) products, for example, then Tariff 1 can be interpreted as a bundled discount (or rebate) in which the list price of each product is \$11, the buyer receives a discount (rebate) of \$10 for purchasing two products, and the buyer receives a discount (rebate) of \$18 for purchasing all three products. It is also equivalent to full-line-forcing, a variant of tying, under certain conditions. In particular, if the buyer's profit from purchasing the bundle of all three products exceeds \$15, its profit from any two products is less than \$12, and its profit from any one product is less than \$11, then the buyer will always take the full line.<sup>12</sup> If instead A, B, and C are different units of a single product, then Tariff 1 can be interpreted as a quantity discount, where the the average price if one unit is purchased is \$11, the average price if two units are purchased is \$6, and the average price if all three units are purchased is \$5. Note that in this instance each object  $i$  is bundled with each other object  $j$ ,  $j \neq i$ , regardless of whether the buyer is also purchasing the third object  $k$ ,  $k \neq i, j$ . This would not be the case under a two-part tariff scheme where conditional on the first unit being purchased, additional units are sold unbundled, although it would still be true from our definition that each object is bundled with each other object over  $X$ , and each object is bundled with the set of all other objects over  $X$ .

Under Tariff 2, A and B are bundled with each other, but C is not bundled with objects A, B, or the set of objects  $\{A, B\}$ . As an example of this scenario, suppose that A and B represent the units of a product labeled 1 and C is a unit of a different product labeled 2. Then Tariff 2 can be interpreted as a quantity discount schedule for product 1 that does not bundle products 1 and 2.

Under Tariff 3, the price of any set of objects is the same as the sum of the prices of the individual objects. There is no sense in which any object is bundled with any other object. However, there

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<sup>12</sup>A more conventional full-line forcing contract might specify a price for each product of \$5 (i.e., = 15/3) and combine it with a contractual requirement that forces the buyer to purchase the supplier's full line of products if it purchases any one of the products. In this case, the contractual requirement is equivalent to increasing the prices of all combinations that do not include the full line by enough that the buyer's best strategy is to take the full line.

is nonlinear pricing if A, B, and C are interpreted as different units of the same product. In this case, Tariff 3 features quantity premia (one unit can be purchased at a cost of \$10, two units can be purchased at an average cost of \$10.50, and three units can be purchased at an average cost of \$11). Thus, under our definition, bundling and nonlinear pricing are different practices. All three tariffs involve nonlinear pricing under some interpretations, but only Tariffs 1 and 2 involve bundling.

### C. The Supplier's Pricing Problem and the Role of Constraints on Bundling

We now describe the supplier's pricing problem. Let  $\mathcal{F}$  be the feasible set of pricing terms given some set of constraints on the seller's ability to bundle. (The specific set of constraints we consider will be described as needed). The supplier's optimal tariff solves<sup>13</sup>

$$\max_{S, p(\cdot) \in \mathcal{F}} p(S) - C(S) \quad s.t. \quad \Pi_B(S) - p(S) \geq \Pi_B(S') - p(S') \quad \text{for all } S' \subset X. \quad (2)$$

The supplier's choice of  $S$  and  $p(\cdot)$  given  $\mathcal{F}$  is an integer-programming problem with a potentially large number of constraints that arise from the buyer's freedom to select from every possible subset of the objects offered for sale. To see how many constraints there are, let  $N(X)$  denote the number of objects in  $X$ . Then, the supplier must consider which of  $2^{N(X)} - 1$  constraints would bind under every possible choice of objects to sell and every possible price schedule defined over  $2^{N(X)}$  possible combinations. With ten objects, for example, there are more than one thousand constraints.

In one special case, however, the supplier's pricing problem has a simple solution. This occurs when the supplier's ability to bundle its objects is unrestricted, i.e., when the feasible set of tariffs is given by  $\mathcal{F} = \{p(\cdot) \mid p : 2^X \rightarrow R_+\}$ . In this case, the supplier can eliminate all constraints in (2) with the exception of the buyer's participation constraint,  $\Pi_B(S) - p(S) \geq 0$ , with an all-or-nothing tariff that charges the buyer prohibitively high prices for all purchases other than  $S$ . In particular, the supplier can use an all-or-nothing tariff to induce the buyer to choose the joint-profit maximizing set of objects  $S^E \equiv \operatorname{argmax}_S \Pi_B(S) - C(S)$ , and it can collect the buyer's profit and earn the associated maximized joint profit by setting  $p(S^E)$  equal to  $\Pi_B(S^E)$  and producing  $S^E$ .<sup>14</sup>

The more interesting cases arise when the supplier's ability to bundle is restricted in some way. The effect of such restrictions on the equilibrium outcome is the topic of the next three sections.

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<sup>13</sup>Formally, the solution is a subgame-perfect equilibrium to a two-stage game in which the seller offers a schedule of prices for all possible combinations of objects, and the buyer makes its optimal purchases.

<sup>14</sup>Varian (1989, Section 2.2) proved this result for the special case in which each object is a different unit of the same product. Here we extend his result to the case of any relationship among the objects.

## II. Pricing When Bundling is Infeasible

We now assume that bundling is infeasible. We first show how this restriction affects prices and alters the supplier's pricing problem. We then characterize profit-maximizing prices and output and explore the efficiency properties of the equilibrium tariffs under this restriction.

### A. How the Inability to Bundle Alters the Supplier's Problem

Under the classic definition, the inability to bundle means that the buyer's total payment for the two objects is the sum of the prices of the individual objects. We now establish that an analogous relationship holds for any arbitrary set of objects when the supplier cannot bundle under our more general definition.

Let  $S' = \{s_1, s_2, \dots, s_{N'}\}$  be a set of objects in  $Z$ . A necessary condition for  $S'$  to be unbundled over  $Z$  is that  $p(S + S') = p(S) + p(S')$  for all  $S$  in  $Z$  that do not overlap with  $S'$ . (This follows from Definition 2 when  $S'' = \emptyset$ .) In particular, it must be true that

$$\begin{aligned} p_{s_1} &= p(S') - p(S' - \{s_1\}), \\ p_{s_2} &= p(S' - \{s_1\}) - p(S' - \{s_1, s_2\}), \\ p_{s_3} &= p(S' - \{s_1, s_2\}) - p(S' - \{s_1, s_2, s_3\}), \\ &\vdots \\ p_{s_N} &= p(S' - \{s_1, s_2, \dots, s_{N'-1}\}) - p(S' - \{s_1, s_2, \dots, s_{N'}\}). \end{aligned}$$

Summing the  $p_i$  over all  $i \in S'$  and using  $p(S' - \{s_1, s_2, \dots, s_{N'}\}) = p(\emptyset) = 0$ , we have  $\sum_{i \in S'} p_i = p(S')$  for all sets  $S'$  that are not bundled over  $Z$ . If there is no bundling over  $X$ , then  $\sum_{i \in S'} p_i = p(S')$  for all  $S' \subset Z$  and all  $Z$  including  $Z = X$ . We summarize this result in the following lemma.

**Lemma 1** *When the bundling of objects is infeasible over  $S$ , the payment for any set of objects purchased in  $S$  is the sum of the prices of the individual objects. That is,  $p(S) = \sum_{i \in S} p_i$ .*

It can be seen using similar logic that the converse of Lemma 1 also holds: if the buyer's total payment for any set of objects purchased is the sum of the prices of the individual objects, then there is no bundling over the purchased set. This comports with the standard definition of bundling in the literature – that bundling arises only when the price of the objects purchased differs from the sum of the individual prices. However, it should be noted that merely observing that the price of the objects purchased differs from the sum of the individual prices does *not* mean that every object

in the purchased set is bundled over that set. It only means that some of the objects are bundled.<sup>15</sup> This distinction will take on added importance in the next section when we allow bundling over some objects but not others, and it is the main reason why we defined bundling as we did — and not by the relationship between the price paid by the buyer and the sum of the individual prices.

Using Lemma 1, the constraints in the supplier’s pricing problem in (2) become

$$\Pi_B(S) - \sum_{i \in S} p_i \geq \Pi_B(S') - \sum_{i \in S'} p_i \text{ for all } S' \subset X. \quad (3)$$

It is easy to see from (3) that the supplier can relax the constraints for all objects in  $X - S$  simply by setting a prohibitively high price (e.g., infinity) on these objects. It also follows from (3) that in order to induce the buyer to purchase some object  $i \in S$ , the supplier’s price for object  $i$  must be less than or equal to the buyer’s incremental gross profit from purchasing it. That is, it must be the case that  $p_i \leq \Pi_B(S) - \Pi_B(S - \{i\})$ . Otherwise, the buyer could gain by dropping object  $i$  from  $S$ .

Nevertheless, the supplier’s pricing problem remains complex even with Lemma 1 because the supplier still faces  $2^{N(X)} - 1$  constraints given the possible choices of objects to sell. The key to solving this kind of problem is to recognize which constraints bind and which do not, and this in general depends on the properties of  $\pi_B$ . The assumption that  $\pi_B$  exhibits declining incremental profit simplifies the problem.

**Lemma 2** *Suppose that over the set  $S \subset X$ , bundling is infeasible and the buyer has declining incremental profit. Then, if the buyer’s net profit from purchasing  $S$  does not increase by dropping any object  $i$  in  $S$ , its net profit also does not increase by dropping any subset of objects in  $S$ . That is, for all  $S^d \subset S$ ,*

$$p_i \leq \Pi_B(S) - \Pi_B(S - \{i\}) \quad \forall i \in S \implies \sum_{i \in S^d} p_i \leq \Pi_B(S) - \Pi_B(S - S^d)$$

*This condition also holds if both inequalities are strict.*

The intuition for this is easy to see in the case of two objects, which the formal proof generalizes. In that case, Lemma 2 implies that if the buyer does not gain by dropping either object alone, then

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<sup>15</sup>The example presented as Tariff 2 in the preceding section is illustrative of how one’s intuition about the meaning of bundling can be misleading. There, the sum of the individual prices for objects A, B, and C exceeds the price of the package of all three, but object C is not bundled with objects A, B, or the package containing A and B over the set  $X = \{A, B, C\}$ . Because objects A and B are bundled with each other, however, a simple comparison of individual prices with the package price of all three would mistakenly conclude that object C is bundled with units A and B or the product consisting of these objects.

it cannot gain by dropping both objects. Note that the buyer’s gross profit loss from dropping either object alone weakly exceeds the price of that object by supposition. The gross profit loss from dropping a second object conditional on dropping the first is even greater by the assumption of declining incremental profit. Therefore, this loss will also exceed the price of the object. This implies that if the buyer does not benefit by dropping one object, it also does not benefit by dropping both objects.

## B. Equilibrium Prices When Bundling is Infeasible

Lemma 2 simplifies the supplier’s pricing problem because it effectively identifies a single constraint on  $p_i$  for each object  $i \in S$ . Since this constraint implies an upper bound on  $p_i$ , and since there is no reason for the supplier to give the buyer more surplus than what is necessary, it follows that  $p_i$  will be set to ensure that the constraint holds with equality. Let  $(p^*, S^*)$  be an equilibrium price vector and set of objects sold when bundling is infeasible. Then we can establish the following proposition.

**Proposition 1** *Suppose the buyer has declining incremental profit over  $S^*$ . Then, in any equilibrium in which bundling is infeasible over  $S' \subseteq S^*$ , the price of each object in  $S'$  must equal the buyer’s incremental gross profit from that object. That is, it must be the case that  $p_i^* = \Pi_B(S^*) - \Pi_B(S^* - \{i\})$  for all  $i \in S'$ .<sup>16</sup>*

Proposition 1 yields two important insights. One is that there exists a unique set of profit-maximizing prices given the set  $S^*$  of objects purchased when the buyer has declining incremental profit. A second insight is that the optimal price of any object in  $S^*$  in this case equals the buyer’s incremental profit from purchasing the object. (Neither of these insights is true if there is increasing incremental profit over any subset of  $S^*$ , as we show in section IV below.) These insights lead to an important corollary for the pricing of objects that are “interchangeable.”

We say that objects  $i \in X$  and  $j \in X$  are “interchangeable” if and only if purchasing object  $i$  would have the same effect on the buyer’s gross profit as would purchasing object  $j$ , given any other combination of purchases the buyer might make. That is,  $\Pi_B(S' + \{i\}) = \Pi_B(S' + \{j\})$  for all  $S' \subset X - \{i, j\}$ . Units that are interchangeable yield the same incremental gross profit for the

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<sup>16</sup>For completeness, it should be noted that the price of objects that are not sold in equilibrium must be high enough that the buyer does not benefit by adding them to  $S^*$  or substituting them for objects in  $S^*$ . At a minimum, therefore, the supplier’s price for any object  $j$  that is not sold must weakly exceed the buyer’s incremental profit from substituting that object for any object in  $S^*$ . This provides a lower bound on the equilibrium prices of the objects that are not sold.

buyer and therefore must be priced the same in equilibrium if purchased. (If one unit were priced lower, the supplier could raise the price of that unit without the buyer dropping it.)

An important example of a case of interchangeable units occurs when the units are of the same *product*, which we define as follows. We say that  $X_k \subset X$  represents the units of product  $k$  if and only if  $\Pi_B(S_k, S'') = \Pi_B(S'_k, S'')$  for all  $S_k \subset X_k$ ,  $S'_k \subset X_k$ , and  $S'' \subset X - X_k$  such that  $S_k$  and  $S'_k$  have the same number of units. In words, the defining characteristic of a product is that conditional on the number of units of the product consumed, the buyer's gross profit is invariant to which units are consumed. This obviously means that any two units of a product are interchangeable. Another example of units that are interchangeable is two single-unit products among a set of symmetrically differentiated products.

**Corollary 1** *Suppose the buyer has declining incremental profit over  $S^*$ . Then, in any equilibrium in which bundling is infeasible over  $S' \subseteq S^*$ , all interchangeable objects in  $S'$  must be sold at the same price, and in particular, all units sold of a given product must be sold at the same price.*

Although an inability to bundle does not prohibit the supplier from setting a different price on each object sold, Corollary 1 implies that when the objects sold are of the same product (e.g., 12-ounce bottles of Coca Cola), this cannot happen in equilibrium. To our knowledge, this is the first formal demonstration that linear pricing is an implication of a supplier's inability to bundle. To foreshadow the analysis that follows, linear pricing is not the cause of the generalized double marginalization distortions we characterize in detail below. Rather, it is a consequence of the supplier's inability to bundle.<sup>17</sup>

### C. Equilibrium Quantities When Bundling is Infeasible

To complete the description of equilibrium, we now determine the supplier's optimal choice of units to sell when bundling is infeasible. By Proposition 1, we know that under declining incremental profit, the price of any object  $i$  that is sold is  $p_i = \Pi_B(S) - \Pi_B(S - \{i\})$ , where  $S$  is the set of all objects sold. We also know that the supplier can relax this constraint for any object it does not want to sell by setting its price to be sufficiently high (e.g., to infinity). It follows that the supplier's maximization problem is then

$$\max_{S, \{p_i\}_{i \in S}} \sum_{i \in S} p_i - C(S) \quad s.t. \quad p_i = \Pi_B(S) - \Pi_B(S - \{i\}), \quad \text{for all } i \in S. \quad (4)$$

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<sup>17</sup>It is well known that linear pricing may also arise from arbitrage among buyers that prevents price discrimination. See, e.g., Tirole (1988). Our point is that even if such arbitrage is not possible (e.g., if the seller can prevent resale), linear price will emerge if the supplier cannot bundle.

We refer to the constraints in (4) as the “incremental profit constraints.” Substituting these constraints into the objective, the supplier’s problem is to choose the set of objects that maximizes

$$\Phi(S) = \sum_{i \in S} \left[ \Pi_B(S) - \Pi_B(S - \{i\}) \right] - C(S). \quad (5)$$

This is in general a hard problem. But progress can be made by recognizing what must be true at the optimum. One condition that must hold is that it must be unprofitable for the supplier to add even one more object, say object  $j \in X - S$ , to the set of objects sold. Let  $\Delta_j \Pi_B(S) = \Pi_B(S + \{j\}) - \Pi_B(S)$  denote the change in the buyer’s gross profit, and  $\Delta_j C(S) = C(S + \{j\}) - C(S)$  denote the change in the supplier’s cost, from augmenting the set of objects  $S$  with object  $j$ . Then, it must be true that

$$\Delta_j \Phi(S) \equiv \Phi(S + \{j\}) - \Phi(S) \quad (6)$$

$$\begin{aligned} &= \sum_{i \in (S + \{j\})} \left[ \Pi_B(S + \{j\}) - \Pi_B(S + \{j\} - \{i\}) \right] - C(S + \{j\}) \\ &\quad - \sum_{i \in (S)} \left[ \Pi_B(S) - \Pi_B(S - \{i\}) \right] - C(S) \end{aligned} \quad (7)$$

$$\begin{aligned} &= \left[ \Pi_B(S + \{j\}) - \Pi_B(S) \right] - \left[ C(S + \{j\}) - C(S) \right] \\ &\quad + \sum_{i \in S} \left\{ \left[ \Pi_B(S + \{j\}) - \Pi_B(S + \{j\} - \{i\}) \right] - \left[ \Pi_B(S) - \Pi_B(S - \{i\}) \right] \right\} \end{aligned} \quad (8)$$

$$= \Delta_j \Pi_B(S) - \Delta_j C(S) + \sum_{i \in S} \left[ \Delta_i \Pi_B(S + \{j\} - \{i\}) - \Delta_i \Pi_B(S - \{i\}) \right] \quad (9)$$

$$= \Delta_j \Pi_J(S) + \sum_{i \in S} \Delta_j \Delta_i \Pi_B(S - \{i\}) \leq 0, \quad (10)$$

where  $\Pi_J(S) = \Pi_B(S) - C(S)$  is the buyer and supplier’s joint profit from selling the objects in  $S$ .

Condition (10) is a necessary condition for profit maximization and has two parts.<sup>18</sup> The first part is the change in the overall surplus created when object  $j$  is added. The second part is the sum of the changes in the incremental profit constraints for each object  $i \in S$  (i.e., the sum of the changes in each  $p_i$ ). The two parts together capture the net effect of adding object  $j$  on the supplier’s profit.

Intuitively, adding object  $j$  to the set of objects sold yields an incremental profit for object  $j$  of  $\Delta_j \Pi_B$ . Because the price of object  $j$  is equal to this amount at the optimum, the net effect

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<sup>18</sup>It is analogous to the supplier’s first-order condition in a bilateral monopoly model with continuous quantities.

on the supplier’s profit holding all other prices constant, after accounting for the cost of unit  $j$ , is thus  $\Delta_j \Pi_B - \Delta_j C = \Delta_j \Pi_J$ . This is the first part of condition (10). The second part of condition (10) accounts for the fact that the prices of all other objects will adjust when object  $j$  is added to the set of objects sold. In particular, when the buyer has declining incremental profit, the change in each incremental-profit constraint will be such that the supplier will have to weakly lower the prices of these other objects to convince the buyer to purchase them.

Although an integrated firm would only care about the first effect, the latter effect clearly mitigates the supplier’s gain from selling object  $j$  (when the buyer has declining incremental profit) and may cause the supplier not to sell object  $j$  even when it would increase joint profits. Since the supplier would also not sell object  $j$  if it decreased joint profits, we have the following proposition.

**Proposition 2** *Suppose the buyer has declining incremental profit over  $S^*$ . Then, if bundling is infeasible, the equilibrium outcome is weakly inefficient. At the equilibrium outcome, the supplier will not add an additional object that would reduce joint profit, but it may fail to sell an additional object that would increase joint profit.*

Proposition 2 points to an inefficiency that arises in a bilateral monopoly setting when bundling is infeasible. It is analogous to the well-known inefficiency that arises in the same setting with one product and continuous quantities when the supplier is restricted to charging linear prices. There, the intuition is often expressed in terms of double marginalization. Because it is constrained to charge linear prices, the supplier is forced to earn its profit by adding a per-unit mark-up to its cost. The buyer also adds a mark-up when reselling to final consumers, and the resulting “double mark-up” yields final prices that are higher than what an integrated firm would charge, i.e., higher than what would maximize the buyer and supplier’s joint profit. Here, we see that what is behind this ‘standard’ intuition is just the notion of declining incremental profits. The inefficiency arises because the supplier must lower the price of each unit it sells whenever it adds to the set of units sold. This is what may cause it to sell fewer units than what would maximize joint profits.<sup>19</sup>

Note that the distortion does not depend on whether the units are homogeneous or differentiated (i.e., on whether the units are of the same product). To see this, it is helpful to rewrite condition (10) in a way that illuminates how the inefficiency relates to the interaction between different products. Suppose the supplier sells  $P$  distinct products, and let  $S_k \subset S$  denote the set of all units

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<sup>19</sup>To put it differently, when the buyer’s incremental profit from selling each unit declines with the number of units sold, the removal of unit  $j$  from the set of units that an integrated firm would sell relaxes each incremental-profit constraint and allows the supplier to increase the price of each unit  $i$ . Thus, removing unit  $j$  may increase the profit of the supplier when bundling is infeasible even though this action would reduce the buyer and supplier’s joint profit.

of product  $k$ ,  $k = 1, \dots, P$ . Let  $N_k = N(S_k)$  be the number of units sold (quantity) of product  $k$ , and let  $i_k$  and  $j_l$  be arbitrary units of products  $k$  and  $l$  respectively. Then, condition (10) can be rewritten as

$$\begin{aligned} \Delta_j \Phi(S) &= \Delta_j \Pi_J(S) + \sum_{i \in S_l} \Delta_j \Delta_i \Pi_B(S - \{i\}) + \sum_{i \in S, i \notin S_l} \Delta_j \Delta_i \Pi_B(S - \{i\}) & (11) \\ &= \Delta_{j_l} \Pi_J(S) + \underbrace{N_l \left[ \Delta_{j_l} \Delta_{i_l} \Pi_B(S - \{i_l\}) \right]}_{\text{Own-Q Distortion}} + \underbrace{\sum_{k \neq l}^P N_k \left[ \Delta_{j_l} \Delta_{i_k} \Pi_B(S - \{i_k\}) \right]}_{\text{Cross-Q Distortion}} \leq 0 & (12) \end{aligned}$$

where (12) follows from (11) because the incremental profit from a unit of a given product is the same for all units of the product. Condition (12) shows that the inefficiency depends on the rate at which the incremental profit from another unit of each product declines with increases in the quantity sold of the same product (the term labeled “Own-Q Distortion”) and the quantities sold of the other products (the term labeled “Cross-Q Distortion”). The Own-Q distortion is analogous to the familiar distortion that arises in the single-product case. The Cross-Q distortion is new. Fully eliminating “double marginalization” requires that both types of distortions be eliminated.

### III. Pricing When Bundling is Feasible Over Some Objects

We now partially relax the restrictions on bundling. We first present a general characterization result for the supplier’s incentive to engage in bundling when bundling is feasible over some objects but not all. We then examine equilibrium pricing under two important special cases: when the supplier can bundle units of the same product but not units of different products, and when the supplier can bundle units of different products but not units of the same product.

#### A. The Incentive to Bundle

We have seen that when bundling is infeasible and the buyer has declining incremental profit, the price of each object is constrained by the buyer’s incremental profit from that object. Thus, when selling the set of objects  $S$ , the supplier faces  $N(S)$  incremental profit constraints that its prices must satisfy. Now suppose that the supplier can bundle objects  $i$  and  $j$  but that it cannot bundle any other objects. The supplier still faces  $N(S) - 2$  incremental profit constraints when setting the prices of objects other than  $i$  and  $j$ . However, by selling objects  $i$  and  $j$  only as a bundle (e.g., with an all-or-nothing offer for  $\{i, j\}$  by setting the prices  $p_i$  and  $p_j$  to infinity), the supplier effectively

replaces the two incremental profit constraints for objects  $i$  and  $j$  with a single incremental profit constraint for the *bundle*  $\{i, j\}$ . Observe that the supplier can always earn at least as much by bundling objects  $i$  and  $j$  this way as it can earn without bundling. The reason is that setting  $p(\{i, j\})$  equal to the sum of the prices charged for objects  $i$  and  $j$  in the absence of bundling is a feasible choice when bundling is permitted over these objects, and the buyer would have no incentive to drop the bundle by Lemma 2.

More generally, suppose the supplier can bundle over each of  $N_Y$  disjoint sets  $Y_1, Y_2, \dots, Y_{N_Y}$ , which are each subsets of  $X$ . If bundling is feasible over each  $Y_k$ , analogous logic seems to suggest that the supplier can do no better than to bundle the units that are contained in each set  $Y_k$  ( $k = 1, \dots, N_Y$ ), and to set prices for all other combinations of units that treat them as unbundled. This logic is correct and is stated precisely in the following lemma.

**Lemma 3** *Suppose the supplier can bundle over each of the sets of units  $Y_1, Y_2, \dots, Y_{N_Y}$ , but cannot bundle over any other sets. Let  $S$  be the units sold in equilibrium under this bundling constraint. In characterizing the price charged for  $S$ , there is no loss of generality from restricting attention to all-or-nothing tariffs that bundle over each set  $Y_k$ ,  $k = 1, 2, \dots, N_Y$ .*

We will use Lemma 3 to describe bundling different subset of objects.

## B. Bundling Units of the Same Product but Not of Different Products

We now allow the supplier to bundle units of the same product but not units of different products. This case might arise if policy authorities prohibit bundling over products but not over units of the same product, or if product bundling is infeasible for any other reason.

### B.1 Under-provision of units

Let  $Y_k$  referenced in Lemma 3 represent the set of units of product  $k$  in the production set  $X$ , i.e.,  $Y_k = X_k$  (where  $X_k$  is the set of units of product  $k$  as defined earlier) for  $k = 1, \dots, P$ . Let  $S$  be the set of units sold. By Lemma 3, we can characterize the equilibrium by assuming that the supplier makes all or nothing offers for  $S_1, S_2, \dots, S_P$ , which are the units in the corresponding sets  $X_1, X_2, \dots, X_P$  that are sold. The constraint on the price of product  $k$  is now the incremental profit constraint for the *bundle* of units of product  $k$ . Thus, the price of product  $k$  is  $F_k = \Pi_B(S) - \Pi_B(S - S_k)$ , and the supplier's total profit is

$$\Phi(S) = \sum_{k \in S} F_k = \sum_{k \in S} \left[ \Pi_B(S) - \Pi_B(S - S_k) \right].$$

At the optimum, it must be unprofitable for the supplier to add an additional unit  $j_l$  to the product  $l$  bundle. The addition of unit  $j_l$  does not affect the buyer's profit in the event it drops product  $l$ , but it does alter its profit by  $\Delta_{j_l}\Pi_B(S)$  when the buyer purchases product  $l$ . Therefore, the price of product  $l$  changes by  $\Delta_{j_l}\Pi_B(S)$ , which changes the supplier's profit by

$$\Delta_{j_l}\Pi_J(S) = \Delta_{j_l}\Pi_B(S) - \Delta_{j_l}C(S).$$

The addition of unit  $j_l$  also changes the buyer's incremental profit from each product  $k \neq l$  by  $\Delta_{j_l}\Delta_k\Pi_B(S - S_k)$ . The effect on the supplier's profit through through the corresponding adjustment in prices is

$$\sum_{k \neq l, k \in S} \Delta_{j_l}\Delta_k\Pi_B(S - S_k).$$

Putting these effects together, the addition of unit  $j_l$  to the product  $l$  bundle changes the supplier's profit by

$$\Delta_{j_l}\Phi(S) = \Delta_{j_l}\Pi_J(S) + \underbrace{\sum_{k \neq l, k \in S} \Delta_{j_l}\Delta_k\Pi_B(S - S_k)}_{\text{Cross-Q Bundle Distortion}}. \quad (13)$$

Condition (13) is much like (12) with two differences. The first difference is that the Own-Q distortion is absent. The reason for this is that the supplier bundles the units of each product, which eliminates traditional double-marginalization. The second difference is that the Cross-Q distortion now occurs through the effect that adding unit  $i_l$  has on the incremental profit from the product  $k$  bundle rather than the incremental profit from each unit of product  $k$ . This difference also arises because the supplier bundles the units of each product. Under declining incremental profit, the Cross-Q Bundle Distortion in (13) is negative, which means that adding a unit to product  $l$  may be unprofitable for the supplier even though it increases joint profit. Thus, the inability to bundle products distorts the quantity of each product.

**Proposition 3** *Suppose the buyer has declining incremental profit over  $S^*$ . Then, if bundling the units of each product is feasible but bundling products is infeasible, the equilibrium outcome is weakly inefficient. At the equilibrium outcome, the supplier will not add an additional unit that would reduce joint profit, but it may fail to sell an additional unit that would increase joint profit.*

The inefficiency here is not “traditional” double marginalization. For example, product-specific fixed fees are feasible, which standard intuition suggests should eliminate the double marginalization distortion. Indeed, the Own-Q distortion—the traditional double marginalization distortion—is

absent. However, a Cross-Q distortion remains. The inefficiency associated with this distortion arises from the same fundamental forces that we have identified as the true source of the double marginalization inefficiency. By restricting the output of any given product, the supplier relaxes incremental profit constraints, allowing it to raise the prices of other products.

## B..2 Under-provision of product variety

The output restriction in Proposition 3 could be a reduction the quantity sold of a given product or the failure to sell even one unit of a product, which would be a reduction in product variety. Thus, the inefficiency we have identified may manifest itself as a reduction in output, variety, or both.

To illustrate this point more cleanly, consider a case where the buyer has constant incremental profit over the units of each product  $k$  up to  $N(S_k)$  units, beyond which the incremental profit from more units is negative. However, the buyer has declining incremental profit over products. An example of this scenario is inelastic demand and constant unit cost for each product, but some degree of substitutability between products. Another example arises when each product is a single unit and incremental profit declines over units.

Suppose the supplier can bundle the units of each product but cannot bundle products. By Lemma 3, we can characterize the equilibrium by assuming that the supplier makes all or nothing offers for  $S_1, S_2, \dots, S_P$ , which are the units sold of the corresponding products  $X_1, X_2, \dots, X_P$ . There is now a close formal analogy between the problem of establishing prices for the discrete bundles  $S_1, S_2, \dots, S_P$  and the problem of pricing individual units examined in Section II. In particular, pricing *products* using product-specific all-or-nothing offers that do not bundle products is analogous to pricing a set of units when it is not possible to bundle units.

Reinterpret  $S$  as any set of *products* sold. By analogy with Lemma 1, the inability to bundle products means that  $p(S) = \sum_{k \in S} p(S_k)$ . By analogy with Lemma 2, if the buyer experiences declining incremental profit over  $S$ , the buyer gains more by dropping product  $k$  than it gains by dropping other products along with product  $k$ . Let  $F_k = p(S_k)$  be the all-or-nothing price for the product bundle  $S_k$ . By analogy with Proposition 1, if there is declining incremental profit over  $S$ , then  $F_k = \pi_B(S) - \pi_B(S - S_k)$  for all products sold. Finally, by analogy to Proposition 2, if there is declining incremental profit over  $S$ , then the firms weakly under-supply products.

This inefficiency — a local under-provision of product variety — is isomorphic to the inefficiency traditionally identified as double marginalization. It arises in the identical analytical framework;

the mathematics and underlying economics are the same. There is no double marginalization in the traditional sense, however, as the demand for each product is inelastic. Thus, a constraint on bundling the units of specific products has no effect in this case. For example, banning quantity discounts, a form of bundling units, would have no effect on the equilibrium outcome. However, the constraint on bundling products distorts variety.

This scenario also shows that constraints on the supplier’s ability to bundle products can be worse than constraints on its ability to bundle the units of the same product. In general, the inefficiency associated with the Cross-Q distortion in (12) may manifest itself as a distortion in output (e.g., a reduction in the number of units of product  $l$ ), a distortion in product variety (products that a joint profit-maximizing firm would sell may be absent from  $S$ ), or both.

### C. Bundling Units of Different Products but Not of the Same Product

Most of the literature on bundling focuses on “bundling products,” which is typically taken to mean offering a per bundle price for bundles consisting of one unit of each product (typically two products) and either linear prices for each product purchased separately (“mixed bundling”) or infinite prices for separate purchases (“pure bundling”). In this section we explain how mixed and pure bundling arise in our framework from different restrictions on bundling and customer preferences. We show that mixed bundling is optimal when it is possible to bundle the units of the different products but not the units of the same product. Pure bundling is optimal when the same condition holds if the buyer prefers to consume products in fixed proportion. We use the same techniques used above to explain why the outcomes under mixed and pure bundling are generally inefficient.

Suppose the supplier can bundle the units of different products but not the units of a particular product. It follows from Corollary 1 that each unit of a given product sold independently (outside of a bundle) must have the same price. In principle, the supplier can establish prices for bundles that include any number of units of each product. However, the following Lemma establishes that the only relevant bundles are those that include at most one unit of any product that is sold.

**Lemma 4** *Suppose the supplier can bundle units of different products but cannot bundle units of any particular product. Then, there is no loss of generality in characterizing the equilibrium outcome from restricting attention to cases where each unit sold of a given product has the same price and all bundles sold include at most one unit of each product sold.*

The intuition for Lemma 1 is the following. Suppose that multiple units of product  $i$  are sold as part of a bundle. Because it is not possible to bundle units of the same product, the buyer's payment does not change if any number of the units of product  $i$  sold in a bundle are purchased separately rather than as part of a bundle. Thus, the buyer's optimal purchase set would not change if the supplier were restricted to including only one unit of product  $i$  in the bundle. In this case, the buyer would simply purchase the other units that it demands separately.

Given Lemma 1, we can restrict attention to cases where the supplier sets a price for each unit when it is sold independently and prices for bundles that each include one unit of each product included in the bundle. Let  $S^*$  be the set of all units sold under this strategy. Let  $I^* \subseteq S^*$  be the set of units that are sold individually (i.e., outside the bundle), and let  $B_1^*, B_2^*, \dots, B_{N_B}^*$  be bundles that are sold. Each bundle consists of one unit of each of two or more products. The supplier's pricing problem now is to put a price on each unit and each of the bundles.

This problem is analytically no different than the pricing problem we have already analyzed. To see this, reinterpret the objects sold as the units that are sold independently and the bundles that are sold,  $B_1^*, B_2^*, \dots, B_{N_B}^*$ . The thought experiment for examining efficiency is the same as it is in cases we have already considered, but now the distortions can arise in three ways: (i) the supplier may choose the wrong set of units to sell independently; (ii) it may choose the wrong set of units to include in a bundle; and (iii) it may choose the wrong number of bundles of a given composition. We can examine each these questions analytically by calculating the incremental profit to supplier from adding a unit to sell independently, adding a unit to an existing bundle, or selling another bundle of a different type. Because the formal analysis is analogous to what has come before, we simply state the result.

**Proposition 4** *Suppose the buyer has declining incremental profit over  $S^*$ . Then, if bundling the units of different products is feasible but bundling the units of particular products is not feasible, the equilibrium outcome is weakly inefficient. At the equilibrium outcome, the supplier will not add an additional unit that would reduce joint profit, but it may fail to (i) sell an additional unbundled unit, (ii) add a unit to a bundle it sells, or (iii) sell an additional bundle that would increase joint profit.*

### C.1 Comparison with mixed and pure bundling

An important difference between our model and the bundling literature is our focus on buyer-specific tariffs. Most of the bundling literature, by contrast, focuses on cases where the same tariff

is offered to more than one heterogeneous buyer. It might be thought that this difference makes analytical comparisons difficult, but some analogies are still possible. Suppose we interpret the buyer in our model as a representative purchaser who buys on behalf of several heterogeneous downstream customers who each demand no more than one unit of each product. Under this interpretation, we can think of the inability to bundle the units of a particular product in our setting as analogous to the inability of sellers in a multi-buyer setting to charge different prices to different buyers. Both constraints limit a type of price discrimination. Under the assumption of declining incremental profit, this constraint yields the inefficiency we have identified. We have not explored the distributional assumptions in the multi-buyer setting that would give rise to declining incremental profit when purchases are interpreted as coming from a representative buyer. This may be an interesting topic for research.

Pure bundling in the multi-buyer literature is a case where the seller cannot offer prices outside the bundle. It is well known that mixed bundling generally dominates pure bundling—this is true because pure bundling is a special case of mixed bundling, so the ability of the seller to engage in mixed bundling must be at least as profitable as pure bundling. For the same reason, mixed bundling dominates pure bundling in our setting as well. Pure bundling is likely to be optimal in our setting only in special cases where it is efficient for the seller to sell the same number of units of each product.

## **C.2 Bundling units of the same product masquerading as bundling products**

The use of the term *bundling* in the literature tends to be reserved for the bundling of products and is not commonly used for bundling the units of specific products. One reason may be that casual observation suggests that the important aspect of bundling that occurs in practice is that price is conditioned on the purchase of other products. But consider the following example, which shows how casual observation can be misleading.

Suppose a supplier sells two products and that the joint profit-maximizing outcome requires selling 10 units of each product. A buyer is willing to pay \$80 for 10 units of product 1 and \$80 for 10 units of product 2, and the buyer is not willing to pay more than \$90 for any number of units of either product. Suppose the supplier charges \$100 per unit for product 1 if the quantity purchased of product 2 is less than 10 units, and it charges \$80 per unit for product 1 if the quantity purchased of product 2 equals or exceeds 10 units. Similarly, the seller charges \$100 per unit for product 2 if the quantity purchased of product 1 is less than 10 units, and it charges \$80 per unit

if the quantity of product 1 equals or exceeds 10 units. Observe that under the assumptions of this example, this tariff achieves the joint profit maximizing outcome.

Does this tariff bundle units of the same product? A casual look might suggest that the answer is no and that this is a counterexample to the claim in Proposition 4 that the inability to bundle units of the same product leads to inefficiency. In particular, the tariff appears to charge a linear price for each product conditioned on purchases of the *other* product. This looks like bundling products.

However, a more careful look shows that this tariff clearly bundles the units of each product. For example, suppose the buyer purchases 10 units of product 2. Then, if the buyer purchases 10 units of product 1 in two chunks of 5 units each, it pays \$1800—\$1000 for the two purchases of product 1, and \$800 for product 2. On the other hand, if the buyer purchases all 10 units of product 1 in a bundle, its payment is \$1600, \$800 for each product. Conditional on purchasing 10 units of product 2, the tariff bundles the units of product 1 because the sum of the prices for purchasing the first 5 units and the last 5 units of product 1 exceeds the price of purchasing all 10 units at once. Because the tariff bundles units of product 1 conditional on some level of purchases of product 2, the tariff bundles the units of product 1 under our definition.

This example achieves the joint profit-maximizing outcome, bundles both units of the same product and units of different products. If the products are independent in demand, it would be possible to maximize joint profits by only bundling the units of each product and not bundling the units of different products. For example, quantity forcing contracts that charge \$80 for each product conditional on purchasing 10 units of the product and a price able \$90 for any other purchase would also maximize joint profits. If the products are gross substitutes, it would not be possible to achieve the joint profit maximizing outcome without bundling both the units of each product and the units of different products.

The purpose of this simple example is to show that the nature of bundling is subtle. What is required here is the bundling of units, but the tariff in the example gives the appearance of bundling products.

## IV. Increasing Incremental Profit

Propositions 1-4 all rely on declining incremental profit, which is related to the the idea that the units are substitutes in derived demand. We now consider incentives for bundling and the effects of

constraints on bundling when there is increasing incremental profit (related to complements) over some or all units.<sup>20</sup>

### A. Increasing Incremental Profit Over All Units

We first consider the case of increasing incremental profit over all units. An example of this scenario would be complementary products purchased as single units.

Suppose that bundling is infeasible. By Lemma 1, the total payment for any set of units is the sum of the prices of the individual units, i.e.,  $p(S) = \sum_{i \in S} p_i$  for all  $S$ . As before, in pricing each unit, the supplier faces  $2^{N(X)} - 1$  constraints for each  $S$ , which are given in (3). The problem is simplified by the following result, which exploits a property of increasing incremental profit.

**Lemma 5** *Suppose that the buyer's incremental profit is increasing over  $S$  and that bundling is infeasible. Suppose further that the buyer's net surplus from purchasing  $S$  is nonnegative. Then there exist prices  $p'_1, p'_2, \dots, p'_{N(S)}$  for the units in  $S$  that sum to  $p(S)$  such that the buyer does not benefit from dropping any subset of units of units in  $S$ . That is for all  $S^d \subseteq S$ ,*

$$\Pi_B(S) - p(S) \geq 0 \implies \sum_{i \in S^d} p'_i \leq \Pi_B(S) - \Pi_B(S - S^d).$$

We illustrate the main idea with a two-unit example that is generalized in the formal proof. Suppose  $S$  consists of two units, 1 and 2, and that the buyer's profits from different combinations of the units are  $\Pi_B(\{1\}) = \Pi_B(\{2\}) = 10$  and  $\pi_B(\{1, 2\}) = 25$ . This is an example with increasing incremental profit, as  $\Pi_B(\{1, 2\}) - \Pi_B(\{i\}) = 15 > 10 = \Pi_B(\{j\})$ ,  $i, j = 1, 2$ ,  $i \neq j$ . If  $p_1 + p_2 \leq 25$ , then the buyer earns nonnegative profit from purchasing both units. The buyer's gross incremental profit from each unit is 15, and the sum of the incremental profits exceeds the total profit from purchasing both units. Therefore, there exist prices for each unit that are less than the incremental profit of the unit and sum to the total profit from purchasing both units.

One way to construct such prices is to set the price of unit 1 equal to the buyer's profit from selling only unit 1, and set the price of unit 2 equal to the buyer's incremental profit from combining unit 1 with unit 2. These prices extract the buyer's surplus from selling both units. This procedure generalizes. For any arbitrary ordering of any set of units  $S$ , one can construct the prices  $p'_1, p'_2, \dots, p'_{N(S)}$  in Lemma 5 by setting the price of the first unit equal to the buyer's profit from

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<sup>20</sup>If units are gross complements in derived demand, then the buyer experiences increasing incremental profit. However, the converse is not true.

selling only that unit, the price of the second unit equal to the buyer's incremental profit from combining the second unit with the first, and so on for all units in  $S$ .

Lemma 5 implies that when increasing incremental profit holds and bundling is infeasible, the only relevant constraint is the participation constraint,  $\Pi_B(S) - p(S) \geq 0$ . It follows immediately that the supplier can induce the fully integrated outcome and capture the fully integrated profit by offering  $S^E \equiv \operatorname{argmax}_S \Pi_B(S) - C(S)$  along with prices such that  $p(S^d) \leq \Pi(S^E) - \Pi(S^E - S^d)$  for all  $S^d \subset S^E$ . Therefore, the supplier does not gain from bundling when the buyer has increasing incremental profit.

**Proposition 5** *Suppose the buyer has increasing incremental profit. Then the supplier can achieve the joint profit maximizing outcome with or without bundling. Constraints on bundling do not affect the equilibrium outcome.*

Proposition 5 in combination with our earlier results shows that the efficiency of the equilibrium tariff depends on two factors: whether incremental profit is increasing or decreasing, and the feasibility of bundling. If incremental profit is increasing over all units in  $X$ , then bundling is irrelevant in our model. If incremental profit is decreasing over all units, then the price schedule is efficient if bundling is unrestricted but it is generally not efficient otherwise.

## B. Complementary Systems — A Hybrid of Increasing and Decreasing Incremental Profit

In many real world scenarios, the buyer has increasing incremental profit over some units but decreasing incremental profit over others. An example is a system comprised of complementary products, which we now consider. In particular, assume that  $X$  consists of  $P$  perfectly complementary products, the units of which are contained in the sets  $X_1, X_2, \dots, X_P$ . By perfectly complementary, we mean that one unit of the system requires one unit of each product, two units of the system requires two units of each product, and so on. Define an  $N$ -unit system as a system that consist of  $N$  units of each product.

In this scenario, whether the buyer's incremental profits are declining or increasing over a particular set depends on both the nature of the set considered and the set of other units purchased. For example, suppose that the buyer purchases two units of each of products 2 through  $P$ . Because the products are perfect complements, adding one unit of product 1 yields a system that consists of one unit of each product, and the buyer's profit is that of a one-unit system. Now suppose

the buyer adds a second unit of product 1. The buyer's profit is then that of a two-unit system. A natural assumption is that the buyer's incremental profit declines with the size of the system. In this case, conditional on purchasing two units of each of products 2 through  $P$ , the buyer's incremental profit from the second unit of product 1 is less than the incremental profit of the first unit. Now suppose the buyer adds a third unit of product 1. Because the products are perfect complements, the incremental profit from that unit is zero, as the buyer has only two units of each of the complementary products, and therefore a two-unit system. We see that conditional on the units purchased of products 2 through  $P$ , the buyer has declining incremental profit over the units of product 1.

On the other hand, incremental profits are increasing over some sets that contain the units of different products. Suppose that the buyer purchases one unit of each of products 1 and 2, and two units of products 3 through  $P$ . Then the incremental profit from a second unit of product 1 is zero, but the incremental profit from a second unit of product 2 conditional on having added a second unit of product 1 is positive, as it increases the size of the system from one unit to two.

Although the buyer's incremental profit declines over some units and increases over others, we can use our previous results to establish the effects of restrictions on bundling. We do this by transforming the problem in a way that makes it more manageable.

Observe that the buyer's gross profit is determined solely by the size of the complementary system. The buyer would never pay a positive price for any unit or collection of units that does not complete a system. Group the units in  $X$  into packages that each contain a single unit of every product. When bundling is infeasible, and the price of each package equals the sum of the prices of the individual units in the package, then the relevant prices to the buyer are the package prices, i.e., the sum of the prices over units in the package. For any given package price, the composition of the individual prices that sum to a given package price is irrelevant. There is no loss in generality, therefore, from redefining  $X$  so that the "units" in  $X$  are packages that each constitute one unit of the system. We will refer to units in this redefined  $X$  as "system units." The supplier's pricing problem when bundling is infeasible is to put a price tag on each system unit.

Given this transformation, the results in Section II are applicable. Suppose bundling is infeasible. Condition (10) must hold at the optimum, with the units interpreted as system units. If the buyer's incremental profit is declining in the size of the system, then the supplier will never sell more than the joint profit-maximizing number of units, and it may sell fewer. Formally,

**Proposition 6** *Suppose  $X$  consists of  $P$  perfectly complementary products, such that a package of*

*units consisting of  $N$  units of each product constitutes an  $N$ -unit system. Suppose the buyer has declining incremental profit over units of the system. Then, in any equilibrium in which bundling is infeasible, the outcome is weakly inefficient. The supplier may fail to sell an additional system when doing so would increase joint profit.*

Proposition 6 implies that restrictions on bundling typically leads to double marginalization even in the case of perfect complements, which exhibit increasing incremental profit over some units. This occurs when the buyer has declining incremental profit over system units, which is a natural case.

Next, suppose that the supplier can bundle the units of each product but not the units of different products. We now dispense with the transformation of  $X$  into system units and interpret it for the moment as the set of all units. By Lemma 3, when it is feasible to bundle the units of each product, there is no loss in generality from restricting attention to all-or-nothing tariffs that charge a price  $F_i$  for a specific number of units of product  $i$  and a prohibitively high price for any other number of units of product  $i$ . Let  $N^E$  be the number of units of each product sold that maximizes joint profit, and let  $F_i$  be the price for  $N^E$  units of product  $i$ . We can now transform the problem by interpreting the objects of analysis as the  $P$  collections of  $N^E$  units of each product. The supplier's pricing problem is to charge unbundled prices for each of  $P$  complementary objects, which together comprise an  $N^E$ -unit system. The solution to this problem has already been described in Proposition 5. The supplier can achieve the joint profit-maximizing outcome.

Alternatively, suppose that the supplier can bundle the units of different products but cannot bundle the units the same product. The same reasons discussed in Section III.C, there is no loss of generality in considering bundles that contain at most one unit of any product. But such bundles comprise complementary systems. Because the supplier cannot bundle units of the same product, it cannot bundle the objects that each consist of a complementary system. In this case, the solution is weakly inefficient.

**Proposition 7** *Suppose that  $X$  consists of  $P$  perfectly complementary products, such that a package of units consisting of  $N$  units of each product constitutes  $N$  units of a system. Suppose further that the buyer has declining incremental profit over units of the system. If the supplier can bundle the units of the same product, then the equilibrium outcome maximizes joint profits. If the buyer can bundle units of different products but cannot bundle units of the same product, then the outcome is weakly inefficient. The supplier may fail to sell an additional system when doing so would increase*

*joint profit.*

## V. Conclusion

We have provided a general definition of bundling that encompasses the bundling of two or more objects over sets of three or more objects. Our definition allows objects to be units of the same product, different products, or both. Such bundling encompasses a range of controversial pricing practices that have drawn antitrust scrutiny in recent years. By nesting these practices in a common framework, we have been able to analyze their microfoundations.

In doing so, we have identified an efficiency motivation for tying and bundling that has been missed in the economic literature. Our analysis shows that the inefficiency in distribution channels known as “double marginalization” arises in a much broader set of circumstances than is generally recognized. It arises in buyer-specific contracting settings whenever two conditions hold: (i) the buyer experiences declining incremental profit over the objects it purchases from the seller, and (ii) the seller is limited in its ability to bundle the objects. If the objects are units of a single product, the inefficiency is Spengler’s double marginalization. If the objects are units of different products, the inefficiency is qualitatively the same as Spengler’s double marginalization.

We find that bundling different products—more precisely, bundling the units of different products—can have efficiency benefits analogous to the effects of eliminating double marginalization. Although this efficiency is a widely acknowledged in the single-product case, it has not been recognized as far as we know in any antitrust case on tying or bundling.

Our findings raise questions about the disparate legal treatment of different forms of tying and bundling under antitrust laws. On the efficiency side of the ledger, our results suggest that when contracts are buyer-specific, non-requirements tying, bundled discounts, and volume discounts have similar efficiency benefits.

Concerns about tying and bundling relate to the potential for these practices to harm competition. Our analysis abstracts from competition issues to focus on primitives that motivate bundling. An area for further research is to incorporate competition into a framework like ours and weigh the benefits we identify against any anticompetitive harms.

## Appendix

Lemma 1, Lemma 4, and Propositions 2-7 are proven in the text. In this Appendix, we prove Lemmas 2, 3, and 5 and Proposition 1.

*Proof of Lemma 2.* Let  $S^d = \{s_1, s_2, \dots, s_{N^d}\}$  be some set of units dropped, with  $i \in S^d$ . Under the supposition that  $p_i \leq \Pi_B(S) - \Pi_B(S - \{i\})$  for all  $i \in S$  and the assumption of declining incremental profit, we have

$$\sum_{i \in S^d} p_i \leq \sum_{i \in S^d} [\Pi_B(S) - \Pi_B(S - \{i\})] \quad (14)$$

$$\begin{aligned} &\leq [\Pi_B(S) - \Pi_B(S - \{s_1\})] \\ &\quad + [\Pi_B(S - \{s_1\}) - \Pi_B(S - \{s_1, s_2\})] \\ &\quad + [\Pi_B(S - \{s_1, s_2\}) - \Pi_B(S - \{s_1, s_2, s_3\})] \\ &\quad \vdots \\ &\quad + [\Pi_B(S - \{s_1, s_2, \dots, s_{N^d-1}\}) - \Pi_B(S - S^d)] \end{aligned} \quad (15)$$

$$= \Pi_B(S) - \Pi_B(S - S^d). \quad (16)$$

The inequality in (14) follows from the supposition that  $p_i \leq \Pi_B(S) - \Pi_B(S - \{i\})$  for all  $j$ . The inequality in (15) is true because each of the  $N^d$  terms in square brackets in the sum is weakly smaller than each of  $N^d$  terms in square brackets in (14) by the assumption of declining incremental profit. Condition (16) follows from canceling terms in condition (15). Q.E.D.

*Proof of Lemma 3.* Let  $(S^B, p^B(\cdot))$  be any solution to (2) when  $\mathcal{F}$  is the set of all tariffs that may bundle over each of the sets  $X_1^B, X_2^B, \dots, X_{P_B}^B$ . Let  $S_i^B \subseteq X_i^B$  be the set of units in  $X_i^B$  that are sold ( $i = 1, \dots, P_B$ ). Because it is not feasible to bundle units in  $X_i^B$  with units in  $X_j^B$  ( $j \neq i$ ),  $p(S^B) = p(S_1^B) + \dots + p(S_{P_B}^B)$ .

Consider an all-or-nothing tariff that, for each collection  $X_i^B$ , charges  $F_i = p(S_i^B)$  if the buyer purchases  $S_i = S_i^B$ , zero if the buyer purchases zero units of product  $i$ , and infinity if the buyer purchases  $S_i \neq S_i^B$  units. This all-or-nothing tariff is feasible, as it bundles the units  $S_i^B$  of each product  $i$ , but does not bundle the units of different products. Observe that it is an optimal strategy for the buyer to purchase the same set of units under this tariff as under  $p^B(\cdot)$ . (The supplier can resolve any buyer indifference by charging a small amount less than  $F_i$  for product  $i$ .) Thus, when it is feasible to bundle the units of the same product but not the units of different products,

there is no loss of generality from restricting attention to all-or-nothing tariffs of this form. Q.E.D.

*Proof of Proposition 1.* Suppose to the contrary that  $p_i^* < \Pi_B(S^*) - \Pi_B(S^* - \{i\})$  for some  $i \in S^*$ . Consider the following three-part deviation by the supplier: (1) raise  $p_i^*$  by a small amount  $\epsilon$  such that  $p_i^* + \epsilon < \Pi_B(S^*) - \Pi_B(S^* - \{i\})$ ; (2) lower price on each of the other  $N(S^*) - 1$  units in  $S^*$  by  $\delta < \epsilon / (N(S^*) - 1)$ ; and (3) charge a very high price (e.g., infinity) for all units not in  $S^*$ . The first component of the deviation raises the price of unit  $i$  by a sufficiently small amount that the buyer's net incremental profit from adding unit  $i$  to its purchase set remains strictly positive. The second component lowers the average price of the other units in  $S^*$  by less than the increase in the price of unit  $i$ . By construction, therefore, this means that if the buyer continues to purchase all units in  $S^*$ , the supplier's deviation will be profitable. The third component ensures that the buyer's net profit will strictly decrease if it replaces any unit in  $S^*$  with any unit  $j \in X$  that is not in  $S^*$ .

The supplier's deviation is profitable, as the buyer will continue to purchase the units in  $S^*$ . The buyer's profit would fall if it responded to the deviation by either adding units not in  $S^*$  or replacing any units in  $S^*$  with units not in  $S^*$  because the prices of units not in  $S^*$  are set too high. Its profit would fall if it dropped some set of units other than  $i$  because it was not profitable to drop them before the deviation and their prices have been strictly reduced. The buyer's profit would also fall if it dropped only unit  $i$  because the new price  $p_i^* + \epsilon$  is less than the buyer's incremental gross profit  $\Pi_B(S^*) - \Pi_B(S^* - \{i\})$  from purchasing  $i$ . Finally, under declining incremental profit, the buyer's profit would fall if it dropped unit  $i$  in combination with one or more other units in  $S^*$  (Lemma 2). This establishes that  $p_i^* = \Pi_B(S^*) - \Pi_B(S^* - \{i\})$  for all  $i \in S^*$  in any equilibrium.

It should be clear that the price of units that are not sold in equilibrium must be high enough that the buyer does not benefit by adding them to  $S^*$  or substituting them for units in  $S^*$ . At a minimum, the price of any unit that is not sold must weakly exceed the buyer's incremental profit from substituting that unit for any unit in  $S^*$ .<sup>21</sup> We have established the proposition. Q.E.D.

*Proof of Lemma 5.* Suppose that  $\Pi_B(S) - p(S) \geq 0$ . Consider an arbitrary ordering of the units in

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<sup>21</sup>Under declining incremental profit, this condition guarantees that the buyer cannot gain by augmenting  $S^*$  with units not in  $S^*$ . We note that this condition provides a lower bound on the equilibrium prices of products that are not sold. A higher price may be required, depending on the nature of substitution among units in  $X$ .

$S, \{s_1, s_2, \dots, s_{N(S)}\}$ . Assign prices to each unit as follows:

$$\begin{aligned} p_1 &= \Pi_B(\{s_1\}) \\ p_2 &= \Pi_B(\{s_1, s_2\}) - \Pi(\{s_1\}) \\ &\vdots \\ p_{N(S)} &= \Pi_B(S) - \Pi_B(\{s_1, s_2, \dots, s_{N(S)-1}\}). \end{aligned}$$

By construction, the buyer does not gain from dropping unit  $s_{N(S)}$ . The buyer also does not gain by dropping any individual unit,  $s_i, i < N(S)$ . The amount saved by dropping unit  $i$ , the price of unit  $i$ , is the incremental profit from adding unit  $i$  to a smaller set than the  $N(S) - 1$  other units. By contrast, the profit loss from dropping unit  $i$  is the incremental profit of adding unit  $i$  to the other  $N(S) - 1$  units. The profit loss exceeds the savings by the assumption of increasing incremental profit.

Finally, the buyer does not gain from dropping any set  $S^d$  of products. Suppose, for example, the supplier were to drop the set  $S^d = \{s_2, s_5, s_8\}$  when  $N(S) \geq 8$ . That is, the buyer drops the 2nd, 5th, and 8th units added in the sequential pricing scheme set out above. The price savings from dropping  $S^d$  is the sum of the following three components:

- (1a) the incremental profit from adding unit  $s_2$  to the set  $\{s_1\}$ ;
- (1b) the incremental profit from adding unit  $s_5$  to the set  $\{s_1, \dots, s_4\}$ ; and
- (1c) the incremental profit from adding unit  $s_8$  to the set  $\{s_1, \dots, s_7\}$ .

The profit lost from dropping the set  $S^d = \{s_2, s_5, s_8\}$  is the sum of the following three components:

- (2a) the incremental profit from adding unit  $s_2$  to a set containing all other units but  $s_5$  and  $s_8$ ;
- (2b) the incremental profit from adding unit  $s_5$  to a set containing all other units but  $s_8$ .
- (2c) the incremental profit from adding unit  $s_8$  to all other units.

Observe that  $(1a) \leq (2a)$ ,  $(1b) \leq (2b)$ , and  $(1c) \leq (2c)$  by the assumption of increasing incremental profit. Therefore, the buyer does not gain from dropping the set  $\{s_2, s_5, s_8\}$ . An analogous argument can be made for any set of units dropped. Q.E.D.

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